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THE ACOUSTICS OF ORCHESTRAL
INSTRUMENTS AND OF THE ORGAN

THE ACOUSTICS OF ORCHESTRAL INSTRUMENTS AND OF THE ORGAN

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PREFACE

A SERIES of Lectures was given by the author at the
Northern Polytechnic, London, in 1929, under a
scheme endowed by the generosity of the late Martin
White for lectures dealing with the organ. The subject
chosen was the tone production of the organ and other
wind instruments. These Martin White Lectures form
the basis of the first five chapters of this book ; to these a
chapter on strings and one on the orchestra in general
have been added, and are now published in the hope that
the matter therein will appeal to musicians and to scientists
interested in music. The organ being in itself an or-
chestra does not receive a special chapter. The various
departments of it are mentioned at appropriate points
throughout the work.

As the lectures were attended by a number of young
persons in the music trades who could not be expected to
have a deep knowledge of physics, the treatment has been
kept as "popular" as possible, consistent with giving those
with more experience an insight into the contributions of
science to music since Helmholtz' day, a good deal of
which has passed unnoticed by the musical world.

In an Appendix an attempt has been made to show how
the new concept of "acoustic impedance" can be applied
to one important aspect of wind instrument design, i.e. the
size and position of the note-holes.

My best thanks are due to Dr. R. S. Clay for a number
of suggestions, and to Mr. E. J. Irons for checking the
calculations in the Appendix. I am also indebted to a

number of individuals and firms who lent blocks or gave permission for reproduction of photographs: to Messrs. Hawkes and Son for lending the blocks for the frontispiece, and Plates VII, X, XII Fig. 1, XV; to Messrs. Taylor and Co. for the block of Plate XIV; to the Physical Society for the block of Plate IV; to the Royal Aeronautical Society for the block of Plate II; to Messrs. Besson and Co. for the loan of the mouthpieces on Plate VIII Fig. 3; to Messrs. Henry Willis and Co. for the loan of the organ pipes on Plate IX Fig. 1; to Dr. Mary Browning for the films of Plates XI and XVI; to Prof. Miller and the Macmillan Co. for permission to reproduce Plate VI from the former's *Science of Musical Sounds*; to the *Physical Review* for permission to reproduce Plate I; to the *Journal de Physique* and Prof. Carrière for permission to reproduce Plates IV and V; to Dr. Trendelenburg for the photograph of Plate XVII; to the Paris-Orleans Ry. Co. for that of Plate XVIII; and to the Pleyel Co. for that of Plate XIX.

<div align="right">E. G. RICHARDSON.</div>

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CHAPTER I

PRODUCTION AND PROPAGATION OF SOUND FROM A WIND INSTRUMENT

THREE processes are involved in "listening to music." There is first the production of sound by the musical instrument—the source. The sound is then propagated through the atmosphere. Finally, the sound impinges on the ear, which analyses the impression it receives into the sensation of a simple tone or a complex mixture of tones. Each of these processes may be regarded as essential to the resulting impression. There can certainly be no sound without the vibrations of the source; an old experiment in which a sounding electric bell is placed in a reservoir from which the air is exhausted shows that no sound can reach the ear in the absence of a transmitting material medium; while we leave the philosophers to determine whether sound can be said to exist if there is no being endowed with ears present to hear it!

It is a commonplace to say that sound is produced by vibration. The vibrations producing sound are usually too rapid to be seen by the unaided eye, but the appearance of a stretched wire plucked aside at one point and then let go shows that it must be vibrating at the same time as the sound is heard. The essential properties which a body must possess in order that it can vibrate are two: elasticity and inertia. Elasticity in the sense that when it is displaced it tends to return to its position of rest, and inertia in the sense that it overshoots that position of rest and

oscillates to and fro before reaching its position of equilibrium. The first factor in causing vibration is an obvious necessity; no one would try to get sound from a stick of plasticine by pulling one end aside and letting go; but the second factor is not quite so obvious. However, take a violin string stretched above a board and immerse it in glycerine, then try to get a sound from it by plucking. After displacement, it slowly returns to its position of rest without oscillation. The energy it possessed when disturbed has been exhausted in overcoming the friction of the liquid.

The sound-producing parts of an instrument in an orchestra are not isolated. In their motion they drag the air into corresponding vibration, and those particles of air which are near the instrument press in their turn upon the more remote particles. But, because the air also possesses inertia, the whole of the air between the instrument and the ear is not set in motion at once. The more remote particles in this movement lag behind the nearer, causing the particles to be compressed closer together near the source of sound, and the compressed condition to travel out from the source in the form of a wave of compression, with a definite speed depending on the atmospheric conditions prevailing at the time. If a single compression is produced, e.g. by firing a gun, it is followed by a single rarefaction of the air where the particles are momentarily separated to an abnormal extent. When this compression strikes our ear-drums (it may be several seconds later), we hear the gunshot. Each particle oscillates once about its position of rest, it is the compression only which seems to move along bodily as a "progressive wave." A sharp noise such as that of a gun, clapping hands, etc., is known as a "pulse." All musical instruments send out a regular succession of pulses, giving to the ear the sensation of a musical tone.

The relations between the sensation of a simple tone and the vibration of the source are simple, i.e.:

The *intensity* of the sound depends on the extent or amplitude of the vibration.

The *pitch* of the tone depends on the number of vibrations in a given time, being usually expressed by the *frequency* or number of vibrations per second.

A sounding body may vibrate in a number of fashions at the same time, so as to produce tones of different frequency at the same time, blending into a complex note. With certain exceptions (reeds and drums) these possible constituents in the complex note given by a sounding instrument have frequencies corresponding to the numbers 1, 2, 3, 4, 5, etc., though certain of these "harmonics," as they are called, may be absent, and some may be subordinated to others. In musical notation, the series of harmonics of frequencies 1, 2, 3, etc., are known as the fundamental, the octave, the twelfth, etc. The relative proportions of these constituents in the mixture corresponding to a complex note fix the "quality" or "timbre" of a note, and determine the difference, for example, between the sound of a clarinet and of an oboe when both are giving a note of the same pitch.

The natural harmonic series of notes based on a fundamental at two octaves below middle *c* on the piano is exhibited in Fig. 1 up to the eighth harmonic. The seventh of these is only approximately given by the *b♭* marked, since it lies in fact between this note and *a*. This harmonic is accordingly dissonant to the diatonic scale, as are in fact many of the higher members of the series. Underneath the notes are shown the corresponding frequencies (on the New Philharmonic pitch).

The same figure exhibits the Helmholtz musical notation, which will be used throughout this book. In this

notation middle *c* is given a small letter, and so are all the notes above it in its octave. The octave of this *c* is marked *c′* or *c₁*, the double octave is *c″* or *c₂*, etc. The octaves in the bass are given capital letters, beginning with C an octave below middle *c*, CC two octaves below, etc. This method saves considerable printing space in referring to notes on the musical scale.
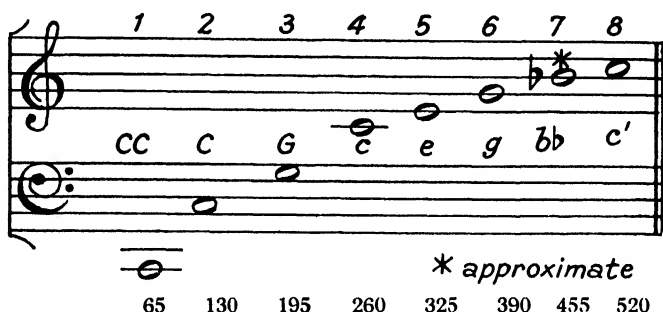
Fig. 1.—Harmonic Series and Helmholtz Notation.

The ear has the power of analysing a complex note into its constituents, a faculty which can be developed with training. Most musicians can pick out the lower constituents (fundamental, octave, twelfth, etc.) in the note given by an instrument. On the generally accepted theory of hearing, the ear contains a series of tuned vibrators which resonate to the corresponding tones falling upon the ear-drums, and which in turn excite corresponding nerves communicating with the brain.

The waves of notes of different pitch must all travel out through the air with the same velocity, otherwise the melody from a band would be unrecognizable at a distance, since a later note might overtake and pass an earlier note, or the bass note of one chord might sound in another chord. With the velocity and the pitch of the note there

FIG. 1.
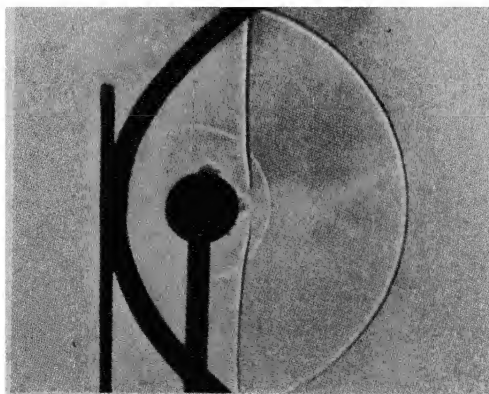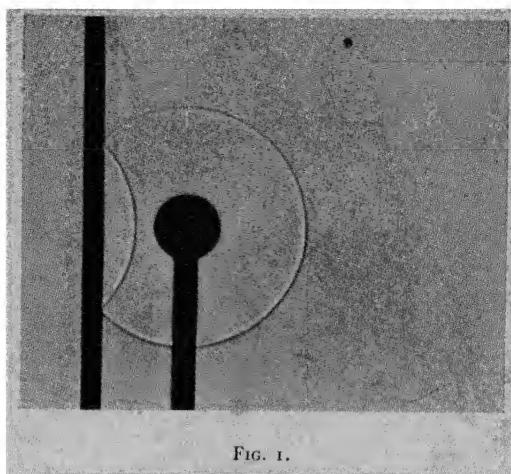
REFLECTION OF SOUND WAVES (Foley).

PLATE I

A.O.I.   17   B

is connected a third quantity, i.e. the wave-length, which may be defined as the distance in air between one compression and the next. The higher the pitch the shorter the wave-length, for the waves then follow each other in rapid succession and are crowded together in space.

If the atmosphere is assumed unlimited in extent the waves travel out in all directions in a straight line from the source. But if the waves reach a solid boundary some or all of the energy in the waves is reflected. Sound in reflection follows exactly the same laws as light, but the phenomenon is on a much larger scale. Light is a wave phenomenon, the waves having an average length of ·005 mm.; whereas the waves of middle c are over a metre in length. One does not usually associate sound waves in the air with visibility, but considerable strides have been made recently in the study of sound waves by actually photographing them. If the vibration which produces the sound is sufficiently vigorous the resulting compression of the air is so intense as to cast a shadow on a photographic plate when the air is illuminated by a sharply defined spark. Usually for such experiments, a single "pulse" is produced consisting of the explosive noise due to the sudden discharge of a large electrical condenser or of an induction coil. A fraction of a second later the air surrounding the source of sound (the spark of discharge) is illuminated by a flash spark, and the shadow of the wave, in its position where the flash caught it, is thrown on a photographic plate behind. By successively increasing the time between the discharge and the illumination, we can trace out the path of the wave, which appears as a circle on the developed plate. Plate I, Fig. 1 shows the reflection of a wave from a plane wooden board, while Plate I, Fig. 2 exhibits the effect of a curved wooden board (of parabolic section) placed as a "soundboard" behind the source. In the

former case the wave is sent back as a wave diverging from an imaginary centre behind the plane, in the latter the reflected wave is plane, and its section, seen on the plate, is a straight line.

In most of the musical instruments we shall have to consider, the principal sound-producing element is a column of air in a tube bounded by wooden or metallic sides. As a simple case let us first consider the column of air in a tube "stopped" at one end by a rigid wall, e.g. a stopped organ-pipe. If a compression travels down such a pipe to the stopped end, it is reflected there as a compression with little loss of energy. But when this reflected wave reaches the open end, reflection again takes place owing to the sudden widening out of the air column and another wave travels down the tube. The resultant effect is that of a series of waves travelling up and down the tube. The pitch of the note given by the tube when incorporated in a musical instrument will depend on the time taken for a compression to travel down the tube and back to the open end, i.e. on the length of the tube.

To demonstrate this important relation, take a glass tube terminated at one end by a water surface (Fig. 2) and open at the other.

If a tuning fork be sounded at the open end, and the tube be gradually· pulled out of the water, a length will be found at which the sound of the fork is strongly reinforced; we have, in fact, "resonance" by the column of air to the note of the fork. When the prong is at x it sends a compression down the tube; in this case, the prong has moved up to y in the same time as the compression has traversed twice the length of the tube, so that the motions of the air and the fork are in step and assist each other. In a complete vibration of the fork, the pulse would therefore traverse four times the length of the tube, and so four times the

length of the tube corresponds to one complete wave-length of the tone.

This is, however, not the only length which will resound to the fork.   If the tube be pulled farther out of the water, another resonant length, nearly three times the first, may be found (Fig. 2*b*), and another, five times the first, etc. In these cases the column subdivides itself into segments
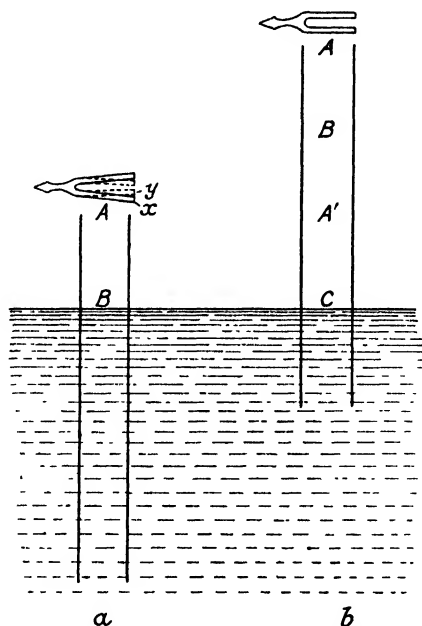
FIG. 2.—Resonant Lengths of a Stopped Pipe.

corresponding to tones which the violinist calls "flageolets" (or harmonics).   The compression in Fig. 2*b* has only to travel as far as B and to be there reflected, other waves of equal pitch run between B and C.   Obviously a barrier could be placed at B without affecting the pitch of the sound.

These segments or loops in the vibrating column can be shown in a still more striking way by employing a sound

of high pitch (and therefore of short wave-length), e.g. a whistle blown with "heavy wind" at the end of a stopped glass tube laid horizontally (Fig. 3).

If pith dust be strewn along the tube and a slight adjustment of the position of the stopper be made to secure resonance, it will then be found that the sand is undisturbed at points corresponding to B or C of Fig. 2, but is violently agitated at the half-way points like A. The first points are called nodes; they are places where the air remains still, although the pressure alternates rapidly many times a second. The latter are called antinodes, and are places where the to-and-fro motion of the air is a maximum, while
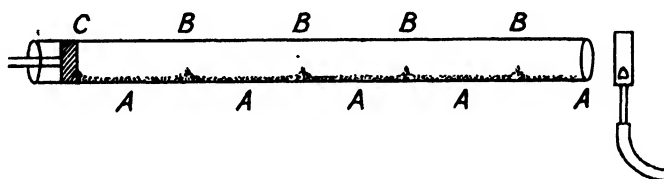
FIG. 3.—Dust Resonance Tube.

the pressure remains constant. Thus the distance between one node and the next antinode is a quarter of the wave-length of the sound; or one may say that the wave-length is equal to two loops or segments.

A stopped end of a pipe is always a node, since the air is prevented from moving as a whole; while an antinode is always found near an open end, as, for instance, the bell of a wind instrument. The lowest or fundamental mode of vibration in a pipe open at both ends is, therefore, that in which antinodes are found at both ends and a node at the centre. Thus we require an open tube approximately twice as long as the stopped pipe to resound to our fork (Fig. 2); for in this case the compression is reflected as a rarefaction at each open end, and the wave travels up and down the tube in the time of a complete vibration, and consequently

the wave-length is only twice the length of the tube. These considerations give us the reason of the well-known fact that a stopped pipe sounds an octave lower than an open pipe of the same length.

In consequence of the possible subdivision of a column
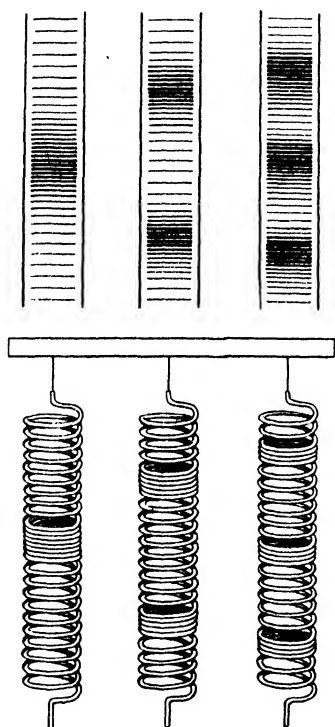
FIG. 4.—Harmonics of an Open Pipe with analogous Motion of a Free Spring.
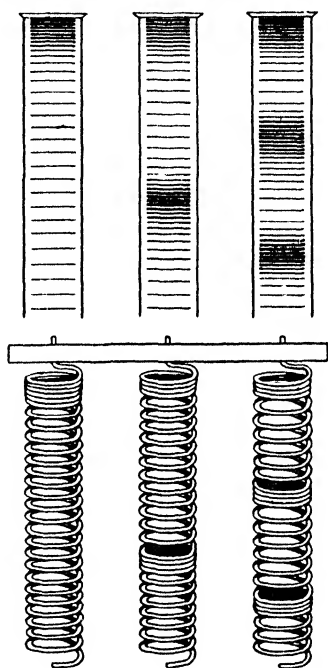
FIG. 5.—Stopped Pipe and analogous Motion of a Spring fixed at one end.

of air into segments we can obtain "harmonics" of the fundamental tone from the same pipe. The open pipe gives theoretically the full series of harmonics as shown for the first three harmonics in Fig. 4. This type of motion can be imitated by setting a freely suspended spring in vertical vibration.

A stopped pipe gives only the odd numbers of the series, fundamental, twelfth, etc., as shown in Fig. 5. This fact is very important for the wood-wind of the orchestra.

Another way to demonstrate the variation of pressure change along a sounding pipe is to apply a little capsule closed by a thin rubber membrane (Fig. 6) to a hole bored in the pipe, or to one of the note-holes of a wood-wind instrument, the other holes being closed by keys or plugs. The "manometric capsule" is affected by changes of pressure. Near a node the membrane is set into violent oscillation, like a drum skin, with a frequency equal to that of the note produced in the pipe; at an antinode it is not
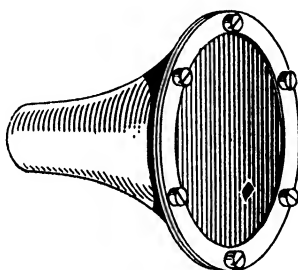
FIG. 6.—Manometric Capsule.

affected. In order to measure the extent of the response of the membrane a very small piece of silvered mirror is stuck eccentrically on to the membrane; if a beam of light is directed on to this mirror it can be reflected on to a transparent scale as a spot of light. When the diaphragm oscillates the mirror, this spot of light is drawn out, owing to persistence of vision, into a band of greater or less width, which can be measured. A graph of the amplitude of pressure fluctuation along a stopped diapason pipe is shown in Fig. 7, obtained by applying such a capsule at a number of points along the pipe.

The writer has also investigated the extent of the aerial

vibration in organ pipes and flutes by passing an electrically heated wire along the tube. The wire is cooled to an extent depending on the movement of the air in its neighbourhood, and this cooling can be measured by the change in the electrical resistance of the wire. This method has the merit that it does not need interference with the pipe by boring or stopping holes in the walls.

The "scale" of the cylindrical tube of a wind instrument exercises two important effects on the tone. Firstly, the quality of an instrument having a wide tube tends to be
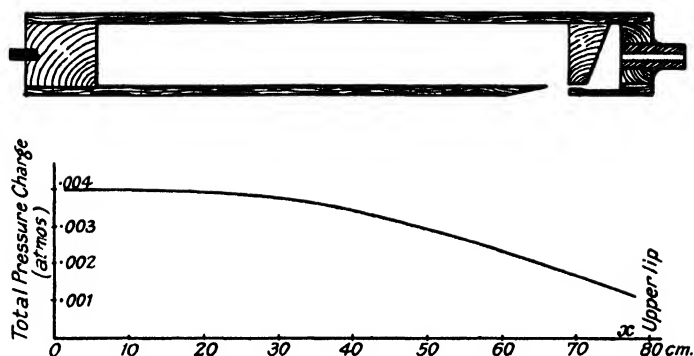
Fig. 7.—Flue Organ Pipe (Stopped) and Variation of Pressure Change along it.

poorer in harmonics than one with a narrow tube; yet, on the other hand, it is easier to "overblow" the latter. In fact, it is often difficult to get the fundamental tone from a narrow pipe, an intermediate node tends to form and to give one of the lower harmonics.

The other important fact which has to be considered in deciding the scale of a tube in a wind instrument is that the antinode at an open end of a pipe is not exactly level with the end of the tube, but lies outside the end by an amount proportional to the diameter of the bore. This means that the "speaking length" of a pipe is not the same

as the length of the wood or metal tube comprising it, but that an "end-correction" amounting to 0·58 of the radius of the tube must be added to the actual length.

From this it results that a narrow tube, besides giving a different quality, has a fundamental tone a little sharper than a wide tube of the same length. One sometimes sees the statement that the difference is due to a difference in the speed of sound in the wide and narrow tubes; but this is erroneous; the speed is unchanged unless the bore of the tube is narrowed down to a millimetre or so, but the wavelength is increased by the end-correction as the tube is made wider.

If the end-correction were the same for all harmonics, these would all be flattened to the same extent, and consequently would remain in tune. Recent research has shown that the end-correction varies a little with pitch. As we go up the musical scale the end-correction first rises a little and then falls. Consequently, if a given pipe of fundamental pitch middle c is sounding its seventh harmonic, this harmonic will be sharp to the fundamental, in one particular pipe, by nine vibrations in 905. If such a pipe gave a whole series of harmonics at once, the harmonics would be seriously out of tune, but, by a saving grace, when such a pipe is sounded, those harmonics which are flat or sharp would be but slightly elicited. It is a property of a sounding system to quench those overtones which are not truly harmonic to the fundamental. This mistuning is greater with wider pipes, a fact which probably explains why a wide pipe gives a meagre retinue of harmonics. But the effect should also vary as between pipes of different pitch built to the same scale.

The special bell-shaped ends of the various instruments will be dealt with under their respective names.

Since a rise of temperature of the air causes an increase

in the velocity of sound, all wind instruments have their pitch raised by this cause. For this reason, players on these instruments warm the air column by breathing through the instrument before using it.

When two sources of sound are giving out tones of nearly, but not quite, equal frequency, a new phenomenon arises which is of much use in tuning two such tones to unison. When the two are out of tune, a slow waxing and waning of the intensity of the sound is heard. This can be illustrated by taking two metal organ pipes of exactly similar dimensions, one provided with a tubular cap which can be moved in telescopic fashion over the main tube of the pipe. Starting with the two in tune, the cap is pulled out over the one pipe so as to increase its length, and therefore to flatten it. As soon as this happens these "beats" in the sound are heard, which follow each other with increasing rapidity as the mistuning gets larger; in fact, the frequency of the beats is just equal to the difference between the frequencies of the two beating tones. The same effect may be produced by keeping the two lengths equal, but warming the air in one by running a flame along the metal outside. As the temperature of the air in the one goes up, the velocity of sound in it increases (*vide supra*), and so does the pitch in like proportion. The warmed pipe being sharper than the cold, the two now begin to beat together.

The same phenomenon may be used to tune together two notes an octave apart. In strings and open pipes the lower note will be accompanied by a good deal of the second harmonic, which should be in tune with the other pipe or string nominally an octave higher. If the second harmonic is elicited powerfully enough, beats will be heard between it and the fundamental of the higher note, unless the tuning is exact.

When the two primary tones are sufficiently far apart for the beats to follow each other fairly fast, the ear will be unable to follow the rapid waxing and waning, and this pulsation will blend into a third "beat-tone," or "difference-tone" as it is more often called, since its frequency is equal to the difference between the two generating tones. This blending will occur when the beats recur at about sixteen times a second, and it is best elicited when the primary
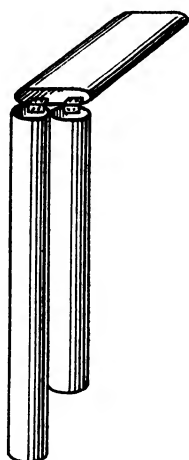
FIG. 8.—Difference-Tone Whistle.

tones are intense, and when the same mass of air is excited by both of them, e.g. from two organ pipes on the same wind chest or two near-by strings on the same soundboard.

For example, if middle c and the g above be loudly sounded on the piano, harmonium, or organ, these having frequencies 260 and 390 respectively on the New Philharmonic pitch, a third tone will be heard of frequency 130 $(390 - 260 = 130)$, which is C. This difference-tone will be more readily detected if the resultant C be sounded before the combination is listened to.

There are, in addition, "summation-tones" which cannot be explained on the beat theory.

This method of employing two pipes to give a third difference-tone low down in the bass forms a cheap method of providing a bass for organs if the presence of the two high-pitched primaries is not objected to. As the primaries may be chosen from among the harmonic series of the difference-tone—in the example above 2nd and 3rd harmonics to the C—this gives a sort of "mixture-stop" for the bass, using very little material and occupying little space. A familiar application is the pair of little unequal Pan pipes (Fig. 8) forming a whistle of the type used by referees and policemen, which has the effect of producing a low difference-tone from two high-pitched pipes which can be carried in the pocket.

CHAPTER II

THE FLUTE

THERE is no aspect of wind instruments to which scientific research has been so directed since Helmholtz' day as the functions of the mouthpiece and embouchure of wind instruments, and since every one admits that tone production has its origin in this part of the instrument, and since the older textbooks have little to say on this important topic, we shall treat this subject in considerable detail, and the results we hope to arrive at will find application later to other instruments besides the flute and its smaller counterpart. the piccolo (see Frontis.).[1]
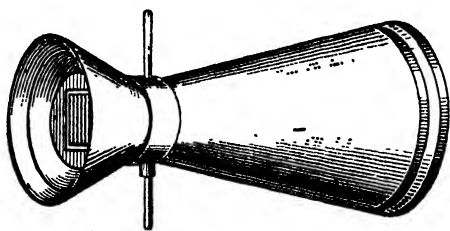
Fig. 9.—Æolian Harp.

Our investigation of the processes which take place in the blast of wind which the player directs across the mouthhole of a flute takes us back to a very ancient musical instrument, if the apparatus is worthy of this title, and one which seems at first sight to have no connection at all with the subject of this chapter: i.e. the Æolian harp. Æolian

[1] The piccolo is shown to a larger scale than the clarinet and oboe beside it.

FIG. 1.—EDDIES IN THE FLOW PAST A ROD.

FIG. 2.—JET ISSUING FROM A SLIT.

PLATE II (after Prandtl).

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harps are still made as toys. In this form they consist of a number of wires of various thicknesses stretched across a framework, on to which the wind is directed by a funnel (Fig. 9).

When the wind blows with appropriate speed, one or more of the wires vibrate across the direction of the wind, sounding one or more of the harmonics natural to the wire. That strings or wires stretched in such a fashion can be made to vibrate by the wind was known from very early times. David had a harp (*kinnor*) which gave out a musical sound in the breeze; but the mechanism of the action was not discovered till 1904. In that year Mr. Mallock noticed that when a stick was held vertically in smoothly running water, eddies with cores parallel to the stick were shed from each side. This, of course, was not an original discovery, but what seems to have escaped previous observation was that these vortices were shed *alternately* from each side of the stick and passed down-stream—as one writer has remarked—"as regularly as the ticking of a clock."

As these vortices were formed, said Mallock, they exerted a periodic push and pull on the stick, in a direction at right angles to the stream, tending to make it vibrate. If instead of allowing the water to flow past a stick, a twig be moved steadily through still water, these vortices can be seen as they are left in the wake, and the periodic force can be felt as a throbbing of the stick. Such a procession of eddies is shown in Plate II, Fig. 1, where they have been rendered visible by flakes in the water flowing from right to left past a rod. The cores of two eddies can be seen to starboard in the turbulent wake, while one lies between these on the opposite side. They are also produced in air behind a rod or wire, and can be then rendered visible by smoke, and if the frequency of the eddies should correspond to one of the natural frequencies of the wire we

hear an Æolian tone as it is called. Even if an Æolian harp is not available, anyone can hear the Æolian tones of telegraph wires as they "sing" in the wind. The frequency of the eddy-production is proportional to the speed of the wind.

To return to our flutes. At the mouthpiece of a whistle or flue organ pipe we have the converse of an Æolian tone. There the moving air debouches through a narrow slit into still air. When moving air rubs against the stagnant air eddies are produced as shown in Plate II, Fig. 2, but their formation cannot be compared to a ticking clock; unless, at most, to a clock which keeps very halting time. But the eddies can be got into proper marching order if the jet of air on issuing strikes against a sharp edge, such as the "upper lip" of an organ pipe, or the distant edge of the mouth-hole in the flute. The pitch of such an "edge-tone," as it is called, goes up regularly with the speed of the air issuing from the mouth, but it also falls as the distance from the slit to the edge increases. For this distance I shall use the organ-builder's term, and call it the height of the mouth.

The existence of such a procession of eddies may be exhibited by admixture of smoke with the air issuing from the slit, or if the whole system be submerged in water by allowing ink to issue from the slit. If the speed is slow we can watch the evolutions of the eddies, but at the actual rate with which these proceed at the mouth of the flute, it is necessary to cinematograph the motion.

Plate III shows a series of such transformations covering a complete period of the motion. These are not actual photographs, but drawings of the appearances seen under a "slow-motion" arrangement.

What may be called the normal production of the vortices occurs in this wise. At the moment when one of the left-handed eddies strikes the edge, as in the first
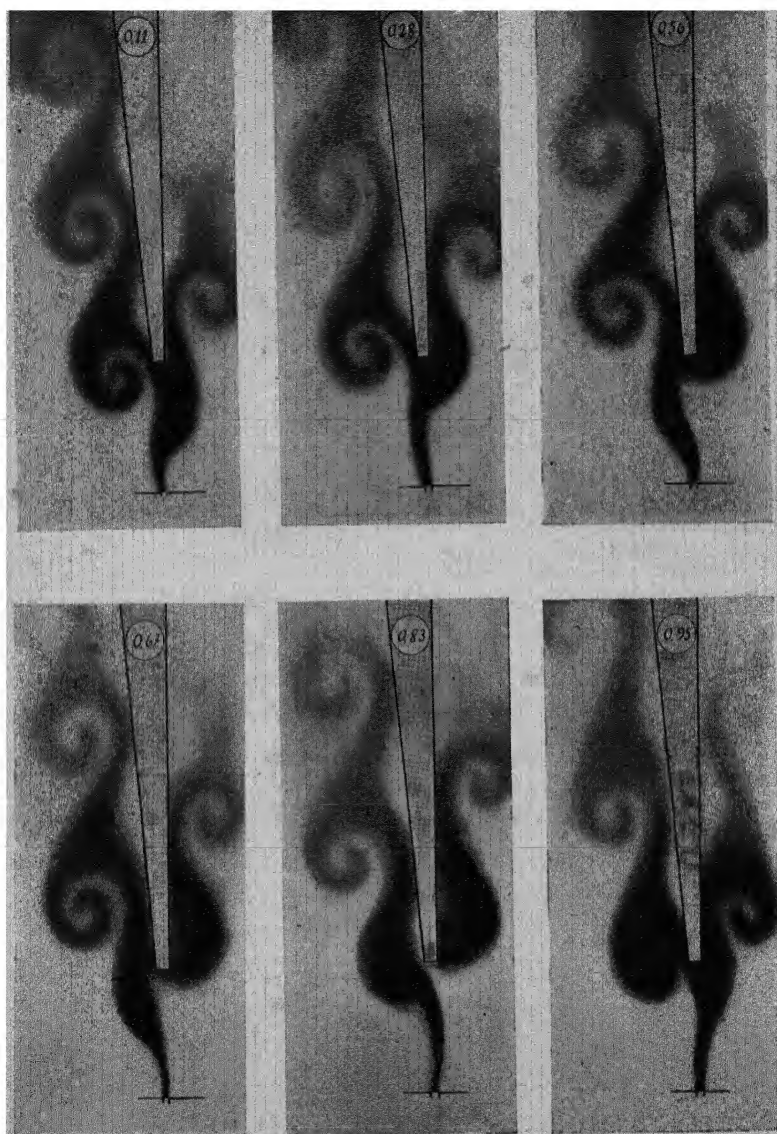
FORMATION OF EDDIES IN AN EDGE-TONE, I.

PLATE III (after Carrière).

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of the series in Plate III, a similar one leaves the slit with one right-handed lying between. Actually, the vortices take time to grow to full size. In this picture the new-born vortex is visible only as a slight leftward bend of the smoke-jet as it leaves the slit, but this embryo vortex grows as it rises, and reaches maturity only as it strikes the edge —in the last of the series. The pitch of the edge-tone is given by the number of vortices in one row which strike the edge per second. Consequently, if the blowing pressure and, therefore, the velocity of the eddies increases the pitch goes up, because the eddies strike the edge more frequently. In fact, we may write:

$$\text{Pitch of edge-tone} = \frac{\text{Velocity of eddy system}}{\text{Height of mouth}}.$$

Moreover, as the velocity of the eddies happens to be just about half that of the main stream issuing from the slit, we may write instead:

$$\text{Pitch of edge-tone} = \frac{\frac{1}{2}\ (\text{Velocity of efflux})}{\text{Height of mouth}}.$$

The truth of this formula may be demonstrated by means of the apparatus shown in Plate IV.

A wedge of brass can be oriented on the traverse of a lathe in various positions relative to a fine vertical slit in the box or wind-chamber B, but the edge of the wedge always keeps parallel to the slit. If I move the wedge away from the slit the pitch goes down; if I increase the blowing pressure the pitch of the edge-tone goes up.

This is the simplest type of edge-tone. In practice, however, especially if the distance from edge to slit is large or the blowing pressure small, the vortices may split this distance into two, three or four segments each containing vortices corresponding to Plate III. Plate V shows some smoke pictures of such cases. That on the left is a repetition of the simplest type, but in the second picture from the

left an additional leftward curl will be observed half-way between the slit and the edge. The third picture has two additional curls to the left, between the big vortex at the edge and the one about to start from the slit. Lying midway between these will be seen corresponding curls to the right. The picture on the extreme right shows a silent jet. No distinct vortices and therefore no sound.

In the state of affairs shown in the second of these pictures, there are four or five vortices in the distance formerly occupied by two or three, but, as before, as each vortex
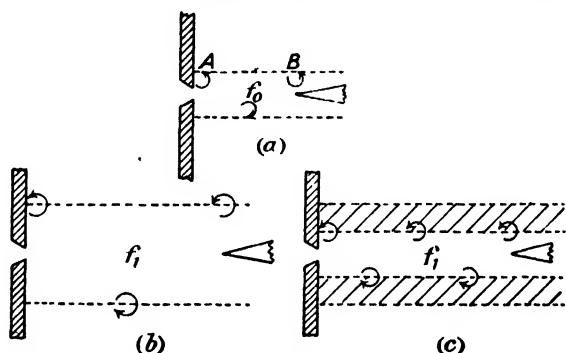
FIG. 10.—Diagrams to illustrate Edge-tone Jumps.

strikes the edge another one sets out from the slit on the same side. A little thought will show that, for the same wind-speed, there are now twice as many striking the edge in a given time as in the former arrangement; the edge-tone is the octave of the former. In the third position there are three times as many striking the edge in the same time, and the edge-tone is a twelfth above the original. Such a jump to the octave, and then to the twelfth, may occur quite suddenly as the vortices rearrange themselves; the tendency to do this will be accentuated if the "height of the mouth" is made large. This can be demonstrated on the edge-tone apparatus of Plate IV.

Fig. 10 shows diagrammatically what happens when the
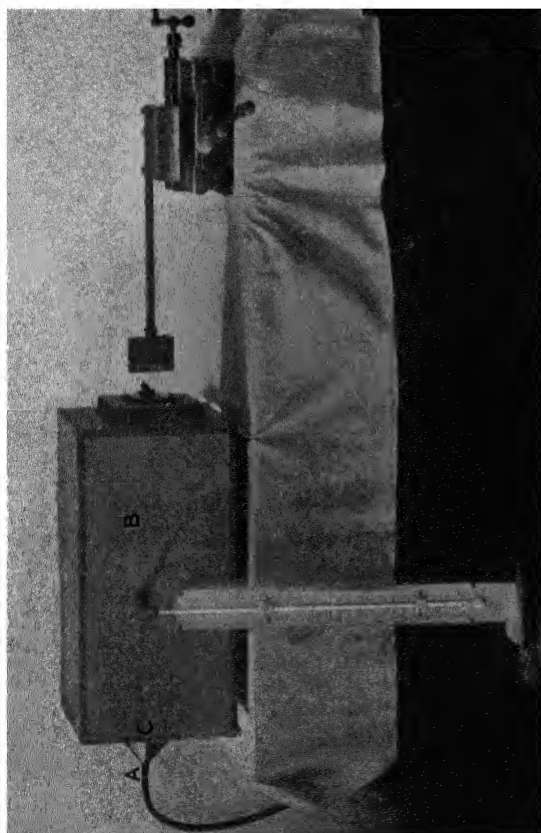
EDGE-TONE APPARATUS (after Benton).

PLATE IV

edge is gradually moved out from a small distance like that in (*a*), to a large distance from the slit as in (*b*). There is a tendency to reshuffling on the part of the vortices to an arrangement like that in (*c*), corresponding to the second of the pictures in Plate V.

When the distance between the successive vortices becomes halved in this fashion, the width of the "avenue" between the two rows also becomes halved. As the edge must separate the two rows to produce a tone, you will appreciate that a flautist can prevent the tone jumping to the octave by blowing so that the edge is a little to one side, within the shaded region of Fig. 10*c*. With the edge in the shaded regions only the simple arrangement of eddies is possible; with the edge in the centre regions, however, a jump may take place. The same precaution is observed on some flute organ pipes, by pushing in a roller (which the organ-builder calls a "beard") from the side, between the edge and the slit. A bearded pipe is shown on Plate IX, p. 67, where the roller can be seen in front of the mouth.

As the older theory of the flute and organ pipe regarded the blast of air at the mouth as a sort of reed which vibrated entirely under the action of the column of air in the pipe, it is very important to realize, for the understanding of the modern scientific theory of such instruments, (1) that the edge-tone may be heard quite independently of any pipe, and (2) that the flute has the physical properties of a coupled system.

Since all musical instruments form coupled systems, I shall elaborate the properties of such systems in some detail. As a simple example, suppose we suspend a rope horizontally, and let two string pendulums of different length hang from two points on the rope. One end I will fix to the wall, and agitate the other with my hand. If I time my movements to synchronize with the natural time-period of

one of the pendulums, the latter builds up a very intense oscillation—this is what the scientist calls resonance, i.e. synchronization of periods of vibration. But even when I move my hand to a different time, the pendulums are forced into a vibration to correspond to the inexorable motion of my hand, although this be one unnatural to them. The lighter they are the more easily are they so forced out of their natural habit. If I let go, however, or become fatigued in the exercise, one pendulum may force its own vibrations on the rope or on my hand. The motion of my hand and the pendulum forms a coupled system, and we see that in coupled systems the race is to the strong, or at any rate to the more massive. The condition for the successful maintenance of a coupled system is, therefore, one partner to predominate and to be able to overcome the other. As the components become equal in power or more loosely coupled, the system becomes more difficult to manage. Examples from other walks of life occur to the mind at once. Most mechanical instruments—using the word mechanical in no derogatory sense—such as the organ, are of the first class, i.e. tightly coupled systems; while orchestral instruments incline a little towards the latter class, and allow of expression of the player's individuality.

The components of the coupled system, the "flute," are (1) edge-tone at the mouth-hole, (2) column of air. For a moment consider the column invariable, of the organ pipe type. The player blows across the mouth producing an edge-tone which, in the absence of the pipe, rises in pitch regularly with the speed of his breath (dotted line of Fig. 11). But the more powerful column of air in the pipe limits the pitch of the system to its natural series (fundamental, octave, twelfth, etc., of p. 15). The result is that the column pulls the feeble edge-tone out of its true pitch, and
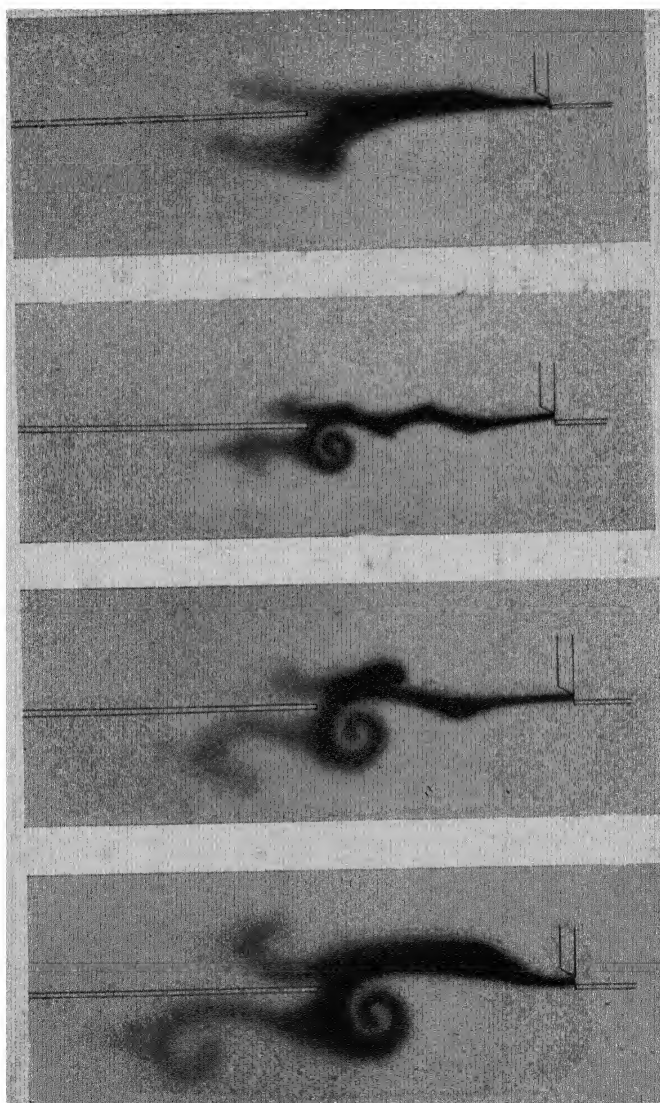
FORMATION OF EDDIES IN AN EDGE-TONE, II (after Carrière).

PLATE V

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the notes of the system are shown by the thick lines; nearly constant at each harmonic except for slight decrease of pitch at lower wind-speed, and increase at higher. The skilful flautist uses these facts to "pull" the instrument a little sharp or flat. The "overblown" octave may be helped in another way: decreasing the "height of the mouth" by pushing the upper lip forward. To exemplify the properties of the coupling, we may suppose the blowing pressure to be continuously increased beyond the normal. As it is increased the natural frequency of the edge-tone rises beyond the fundamental of the pipe, but the latter
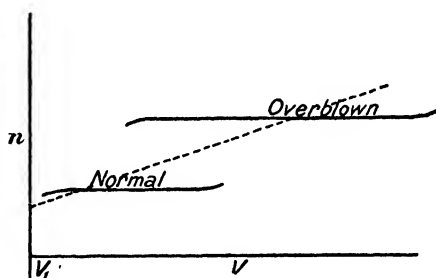
FIG. 11.—Tones of Normal and "Overblown" Flute.

succeeds in forcing its own period upon the edge-tone until the natural frequency of the edge-tone, if isolated, would be nearer to the first overtone of the pipe than to the fundamental. Up to this moment the frequency of the coupled system has remained in the neighbourhood of the fundamental of the pipe, but now a jump occurs to the overtone, both edge-tone and pipe-tone rapidly picking up the new frequency, which they retain with a slight alteration until a jump to the next overtone takes place. Actually the behaviour is more complex in that the overtone may appear before the fundamental has ceased, producing a complex note. The procedure is shown graphically in Fig. 11, where the actual pipe-tones of an open pipe are shown by

thick lines, and the natural frequency of the edge-tones, in the absence of a pipe, by a dotted line.

The converse phenomenon of the "underblown" pipe presents several items of interest. Suppose the pressure to be continuously decreased below normal. Very soon the edge-tone will be so far below the pipe-tone that the latter ceases to sound. But now, the two systems being still coupled, the pipe is still endeavouring to impose one or other of its own tones upon the vortex production at the mouth, and in an endeavour to conform, the edge-tone reduces its pseudo-"wave-length" to one-half or one-third of the height of the mouth, and the same tone is again, but feebly, elicited (cf. Fig. 12).
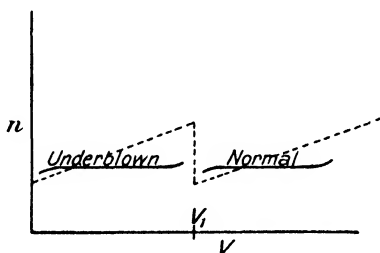
FIG. 12.—Tones of "Underblown" Flute Pipe.

Having now considered the relation between the edge-tones at the mouth and the column of air, we must now turn to the form and possible variations in the latter. Boehm gave the flute its present-day appearance and form. The primitive flute consisted of a cylindrical tube, often a hollowed-out cane, pierced with half a dozen equi-spaced holes all of the same size and depth. The spacing was done by measuring two or three finger-breadths or by placing a marked template over the wood. The holes were then burnt through. When Boehm improved the flute, additional holes worked by levers were already known.

These levers operated additional holes beyond the reach of the fingers.

It is well known that the chief improvement which Boehm brought to the flute was a steady contraction of the bore towards the embouchure, amounting to about one-tenth of the diameter at the cork. It is also known that such a contraction improves the tone, but writers on the flute are at a loss to explain the cause of this improvement. Now it has been found that a widening of the bore of a pipe near an antinode of the note which it is sounding raises the pitch of that note, but near a node the pitch is lowered. The embouchure being open to the air forms an antinode to all notes of the flute ; if all the holes be closed, the open end will be an antinode, and the fundamental tone will be that corresponding to a doubly-open column of the whole length from embouchure to distant end, lowered somewhat by the contraction at the former antinode. But besides this fundamental tone, overtones will be heard due to subdivision of the column into vibrating segments. Their pitch will be lowered, but *not to the same extent*, since some of their antinodes are in the midst of the column where the contraction is less. The second harmonic will be a *little more* than an octave above the fundamental, the third more than a twelfth, and so on. In other words, the series of tones forming the complex note given by such a flute will deviate from the pitch relation 1 : 2 : 3 : 4, etc., the deviation increasing up the scale.

Now, it is an established fact that if, owing to its construction, a system has certain overtones which are flat or sharp to the tones of the true harmonic series, i.e. are inharmonic, *such overtones are but feebly elicited when the system is set in vibration.* Hence, the note of a Boehm is pure in a sense that the retinue of upper harmonics is meagre, partly owing to this cause and partly also to the

fact that these "inharmonics" are out of tune to the true harmonics in the existing edge-tones (cf. Chapter I). Plate VI, Fig. 1, p. 51, shows a record of the vibrations of the flute, when played softly, and shows by the pure and unbroken sinusoidal wave-form that the "note" is almost entirely composed of an isolated fundamental.

Although Boehm's contraction gives the flute its characteristic thinness of tone, yet it has an inconvenience. Suppose the player uncovers in turn the note-holes and so ascends the lowest octave of the flute's range (i.e. from middle *c* to *c'*). As we have explained, he now re-covers all the holes, and changes the edge-tone by altering the "height of the mouth," and the breath-speed, until the column overblows to the next octave, and after this, by more overblowing, to the third octave. But, in virtue of the contraction, as we have seen, the second and third octaves, being overtones of the first, are out of tune to the first octave. To remedy this, Boehm constructed the little chamber beyond the mouth-hole, closed with the adjustable cork. By moving the cork the player can slightly alter the position of the antinode at the embouchure, and he is instructed to get the three D's in the three octaves as nearly in just intonation as possible, by this means. The upper overtones are still out of tune and stifled, and the relation for other notes of the scale is not quite just, but the player can pull the system into tune by working on the edge-tone at the mouth, or by partly covering the mouth-hole with his upper lip. It is, of course, this capability that the performer can modify the pitch of all notes and, to a little extent, their quality, by altering the edge-tone through change of direction and speed of the blast, which raises the orchestral flute as a musical instrument above the level of the primitive pipes or the modern tin-whistle.

One problem remains to be discussed, and that a very

important one, the position of the note-holes. In this connection it is advisable to introduce to the reader a concept which he will not find in Helmholtz, but which has been of considerable use to acousticians in recent years. It is obvious that a narrow hole in the side of a tube resists the motion of air through it, to a greater extent the narrower and deeper it is. Lord Rayleigh defined what he called the "conductivity"—the opposite of resistance— of the orifice as the ratio of its area to its depth, but in measuring its depth, i.e. the thickness of the wood, he pointed out that a little must be allowed for the fact that the hole impedes the motion to a distance a little beyond its faces. In order to get the effective depth of the hole, a length equal to about a third of the diameter of the hole should be added to the thickness of the wood. Then the conductivity (c) of the hole is:

$$c = \frac{\text{area}}{\text{depth} + \frac{1}{3} \text{ diameter}}.$$

The greater this factor is, the less is the "impedance" or hindrance to the motion; but another factor enters, i.e. the inertia of the air, or its tendency to remain where it is. The air has to be oscillated to and fro in the orifice by the sound, as is a flexible piston in a cylinder. If anyone tries to oscillate a piston he will find that the effort required depends on the mass of the piston ($=$ the density of the air), and the frequency of the oscillation ($=$ the pitch of the sound). In this work it is more convenient to use the cycle-frequency (k).

$$k = \frac{2\pi \times \text{frequency}}{\text{velocity of sound}},$$

and we finally define the "impedance" of an orifice as $\dfrac{kda}{c}$, where $d =$ density of the air, $a =$ velocity of sound, c and k are defined as above. This concept of "acoustic impe-

dance" was introduced by the late Prof. Webster in 1919, who also worked out the "impedance" of an open and of a stopped pipe. Sometimes the "admittance" is spoken of; this is the converse of impedance. These two terms—admittance and impedance—are adopted from alternating-current technology.

Now consider an open pipe with a hole of conductivity c bored in its side. In this case the air "admitted" to the atmosphere from the inside of the pipe comes partly from one section and partly from the other, so that we can equate the admittance of the orifice to the sum of the admittances of the two open pipes formed by the two sections of the flute on each side of the hole. From such an equation we can calculate k, which gives us the pitch of the flute, when this particular hole is open and all the others closed. Conversely, if we are given k, we can calculate where to put the hole, and what size to make it for this given pitch, instead of resorting to the present "cut and try" methods of the manufacturer. This and other cases of several holes uncovered will be found worked out in the Appendix, p. 147.

The student who desires to dive deeper into the mysteries of vibrating waves of air may note one particular in which the idea of the conductivity of a note-hole requires modification. The analogy of the steam piston is imperfect, for it has been found that the air does not vibrate as a whole, but that it tends to concentrate its motion into the outer annuli of the hole, and to vibrate to a greater extent near the wall of the hole, to an extent increasing with the frequency. The same phenomenon occurs in the oscillatory electric currents employed on many electric railways. Although the massive iron rails are used to conduct this current, the current crowds so much into the outer "skin" of the rail that, at the frequencies commonly employed, only
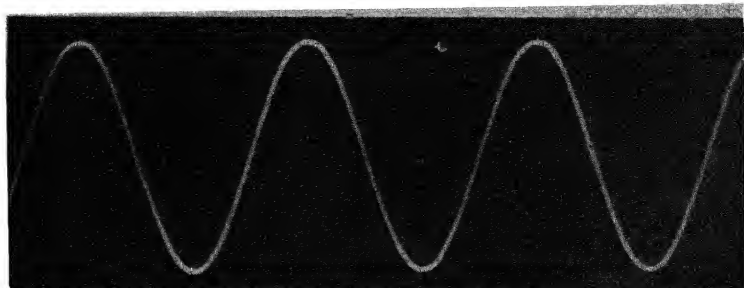
FIG. 1.—WAVE-FORM OF FLUTE.

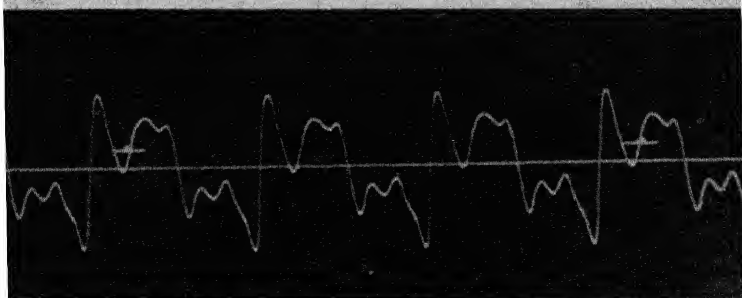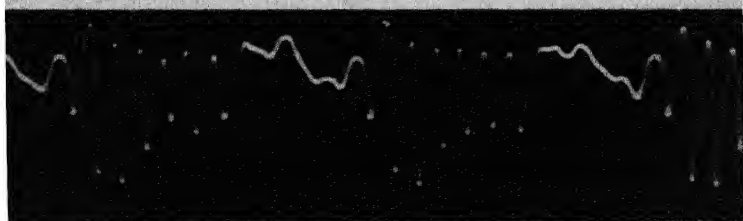FIG. 2.—WAVE-FORM OF CLARINET: CHALUMEAU.

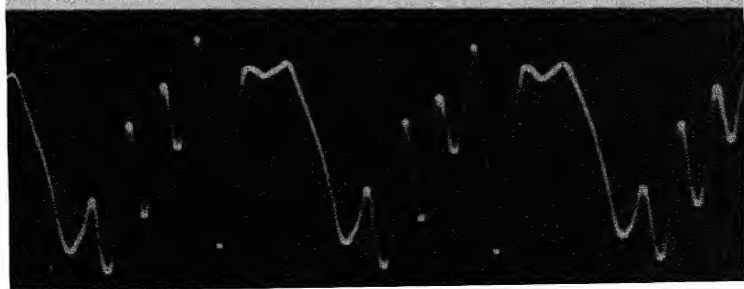FIG. 3.—WAVE-FORM OF CLARINET: UPPER REGISTER.

FIG. 4.—WAVE-FORM OF OBOE.

PLATE VI (after Miller).

about one-eighth of the section acts appreciably as con-
ductor.   In both the acoustic and the electric case this
annular effect increases the resistance, but, at present, there
is no means of calculating this increase, at least in the
acoustic case.

CHAPTER III

REED INSTRUMENTS

IN this class of instrument the coupled system consists of a column of air in a cylindrical or a conical pipe, and a reed or pair of reeds. Since the air in the player's mouth is directly connected to the pipe via the mouthpiece, the system is really a tripartite one, and the size of this cavity (the "boot," in the case of the reed organ pipe) should be strictly adjusted to the note to be produced. Similarly in playing an orchestral instrument, the cheeks may be compressed for ease in producing a high note—more especially is this true of a brass instrument. The player should imagine he is to sing the note required.

First we will take the clarinet. This has a cylindrical tube in three sections terminated at the open end by the bell, and chamfered at the mouthpiece to fit between the player's lips (Frontispiece). To the flat side of the mouthpiece a cane reed is secured by two ligatures. The reed is tapered in thickness from the ligature to the beak (Plate VIII, Fig. 1, p. 63).

The Boehm system of fingering is now applied to the clarinet. When the seven finger-holes are covered and an additional one closed by a key, the complete column of air is in use and the clarinet sounds its lowest note. Nominally this is the same, i.e. E, for all clarinets, but for ease of execution of passages in keys involving several sharps or flats, the orchestral clarinet is made as a transposing instrument. On one instrument, the actual notes are a whole

tone lower in pitch than the written ones; on another, they are three semitones lower. This allows the composer to write, and the performer to play, in easier keys than those being read by the other players.

Since clarinets are now sometimes made of silver, instead of wood, it behoves us to consider what influence, if any, the material of the tube may have on the tone of the instrument. This is a perfectly general question, and the remarks which follow are not to be taken as peculiar to the clarinet. Theoretically, the tone-producing parts consist only of the column of air and embouchure. Regarded in this light, it would be immaterial for the tone of the instrument whether the confines of the air column were of wood, metal, or papier mâché. Experiment shows, however, that if the walls are yielding or sound-absorbent there is a slight lowering of pitch as compared with a rigid wall, but, worse than this, the tone is weak and rapidly damped, for much of the energy of the player is used up in vibrating the walls instead of the column of air. This may be shown by blowing one of those papier mâché "instruments" which are sold for Christmas amusement, and, while doing so, by grasping the "tube" firmly with the hands. The tone gets louder on grasping, for the waste of energy has been reduced. To the extent that silver is more rigid than oiled wood, an improvement of increased efficiency may be expected from a flute or clarinet made of this material (Plate VII).

But there is another effect. No wall is absolutely unyielding, and the more rigid one tries to make it the greater is the possibility of its having marked natural frequencies. Should one of these coincide with a note of the instrument, the silver tube will vibrate or rattle in sympathy. The influence of the tube then is twofold: (1) greater or less damping of the tone, (2) an enhancement of notes in

certain regions of the scale. What is called by Hermann the "formant" of a musical instrument is closely connected with this latter property. The term was first coined in connection with the voice. It has been shown by analysis that in singing certain vowels to the same note, the distinctive characteristics of the different vowels lie in an emphasis of certain harmonics lying in a particular region of the musical scale. This "formant" is different for each vowel. The same phenomenon has been distinguished in the analysis of notes played by different instruments. In the old theory mentioned in the first chapter, the "timbre" of an instrument was uniquely determined by the number and proportions of the harmonics relative to the fundamental. If the same instrument played another note, the timbre would be unchanged, but all the harmonics would be shifted so as to retain the same relation to the fundamental. But on the new formant theory, all harmonics lying within a certain range of pitch would be prominent in the note of the instrument, no matter what the fundamental, and it is this predominant range which characterizes the timbre of the instrument. It is this formant which differentiates, for example, a Stradivarius from a Heath-Robinson, still more a cello from a viola when playing the same nominal note. By analysing "formants" one could determine what range of formant is required to produce a good-toned instrument, though the designing of an instrument to have a given formant is at present not practicable.

Hermann-Goldap has analysed the timbre of examples of instruments and discovered the formants shown in the table opposite.

The last column shows the relative intensity of the fundamental in the complex note to those harmonics lying in the formant. Note that in the oboe and trumpet the former is the weaker (fraction less than one), while in the clarinet

REED INSTRUMENTS

Instrument.	Range of Fundamental Investigated.	Range of Tones always Prominent.	Intensity of Fund. Intensity of Formant.
Oboe	f_1 to f_2	$g\sharp_3$ to $b\flat_3$	0 to 0·48
B♭ Trumpet . . .	b to b_1	$b\flat$ to c_4	0·13 to 0·30
Horn	c_1	$b\flat_1$ to c_3	0·57 to 1
Tenor trombone . .	g to a, and d_1 to f_1	$b\flat_1$ to d_2	0·57 to 0·88
B♭ Clarinet [1] . . .	a_1 to e_2	g_3 to $b\flat_3$	1·32 to 5·0
Flute [1]	d_2 to $c\sharp_3$	f_3 to a_3	1·04 to 3·16

[1] Presumably made of wood.

and flute the fundamental overpowers the formant. It is pointed out that not merely the *range of the formant*, but also the *intensity within that range* relative to the fundamental, together determine the timbre; since the oboe and clarinet which were used gave the same range of formant, but the formant of the oboe overpowers the fundamental, hence its more piercing tone.

The formant is then an absolute pitch theory of timbre, as opposed to the older relative pitch theory. It is too early yet to say that it is definitely established, but Professor Miller in America, using delicate sound-analysing instruments, declares his results to favour it. Probably both the absolute and the relative pitches affect the matter.

The remarks in the preceding chapter on coupled systems apply with equal force to a reed instrument. The reed fulfils the function of exciter, the column of air determines the note produced. In order that the air may predominate over the cane, the mass of air set in vibration must be large, and the coupling must be fairly tight; otherwise the reed will escape from its bondage and vibrate on its own. The "quack" produced by an unskilful player on a badly made instrument is the reed's own inharmonious music. The coupling is loosed if there is a leak between the air

column and the reed, e.g. due to a badly fitting mouthpiece joint. Under such conditions the reed cannot readily be pulled out of its natural frequency, with the result that successively lengthening the column of air will lower the pitch of the instrument only a little, and then it will jump back (cf. the theory of the flute). But with tight coupling another default must be guarded against. If the reed be too stiff and strong, the two parties in the coupled system will pull at sixes and sevens, and cease altogether to function, or at any rate will render the player's task very difficult. Hence the reed should be pared down light and thin. More on this matter in the fifth chapter.

Unlike the flute, the embouchure of a reed instrument is a node of the air's vibration. The experiments of M. Carrière on the coupling between a reed and an air column show that the relative phase of the reed's vibration and that of the column depend on how far they are removed from resonance. When the wave-length of the column of air is longer than that of the reed (which is the usual case with the clarinet and oboe) the air pressure is least at about the same instant when the reed presses on the table and shuts off the air column momentarily from the player's mouth. Air enters the tube therefore when the reed gap is open and the pressure is a maximum.

The clarinet is therefore a pipe closed at one end, and, in view of what was said in the first chapter, even harmonics will be absent from the sound it gives, and it will overblow to the third harmonic (a twelfth above the fundamental). The note-holes on the lower range (called the *chalumeau*) are fixed to give notes from E to $b\flat$. A vent-hole at 6 inches from the beak is then opened to "encourage" an antinode in the upper part of the tube, and the player can then cover the range from b upwards. By special fingering it is possible to reach c_2. This vent-hole need not be so

wide as the note-holes, for it has not to allow free passage of air to and fro, but merely to prevent the pressure within from rising above or falling below atmospheric; for this function a small orifice about $\frac{1}{16}$ inch diameter suffices.

Plate VI, p. 51, shows two of Prof. Miller's records of the wave-form of the tone of a clarinet (Fig. 2) in the lower register, where the wave has additional "kinks" superposed on it, due to the prominent third and fifth harmonics, and (Fig. 3) in the upper register, where the higher overtones in the sound are prominent, and, being a little inharmonic to the true harmonic series, make "beats" with each other, which beats can be seen in the slow waxing and waning —repeated nearly three times in the section of the film reproduced—on the faster vibrations of the tones themselves.

Owing to the small mass of air in vibration, notes from f to $b\flat$ are weak and ineffective. Moreover, some of the upper notes tend to be out of tune with the lower, because the twelfth vent-hole is the same for all notes in the upper register, whereas the position of the extra antinode should vary. The player can remedy this, by shortening or lengthening the free portion of the reed between his lips, or by pressing harder upon the root of the reed, i.e. by working upon the other member of the coupled system. The influence of a bell-shaped end will be discussed under brass instruments, but obviously that influence diminishes when several of the lower note-holes are open. The calculation of the relation between the note-holes and the fingering proceeds in the same way as in the flute (see Appendix).

The oboe differs from the clarinet in two important respects. Firstly, it has two reeds beating together instead of a single reed beating against a rigid table. Secondly, it has a conical instead of a cylindrical tube, the reed being at the apex of the cone. Oboe reeds are much finer and

lighter than those used on clarinets and are bound together by a thread at their roots, and thus placed in a metal staple which adjoins the narrow apex of the conical column of air (Plate VIII, Fig. 2). The reeds normally leave a very narrow ○-shaped orifice at their free ends. When in vibration they alternately open and close this gap; this they do under the action of the player's breath as they are held in his mouth. In consequence of their lightness very little wind is required in blowing the instrument, and the pressure of the lips has not to be adjusted to a nicety.

The note produced by the system is governed by the conical column of air. The sound-producing properties of such a column differ from those of a cylindrical column. In the latter the walls are parallel and we have to deal with *plane* waves advancing in the direction of the axis of the pipe, in the former we have *spherical* waves diverging from or converging on the vertex of the cone. By mathematical analysis too involved to be given here, it can be shown that the harmonics of a conical column *closed* at one end and *open* at the other are the same as those of a cylindrical column *open* at both ends, and of the same length, i.e. the full harmonic series, and not the odd ones only as in the clarinet.

We can show this experimentally by taking a cone, a cylinder open at both ends, and a cylinder closed at one end (Fig. 13). These—the first pair of nearly equal length, the last of half this length—are chosen so that all resound to a fork held over an open end; this is, in fact, their fundamental tone. Now, if I take a fork an octave higher, the doubly-open tube resounds to it, the cone closed at one end also reinforces the fork, but the cylinder closed at one end (clarinet type) remains silent. All three respond to a fork of the twelfth.

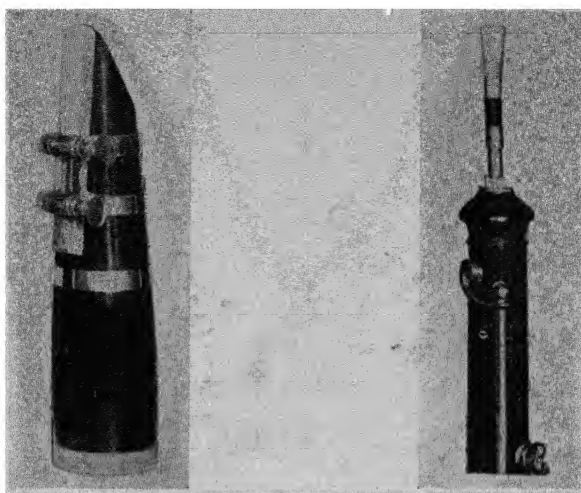The complex note given by the oboe (Plate VI, Fig. 4,

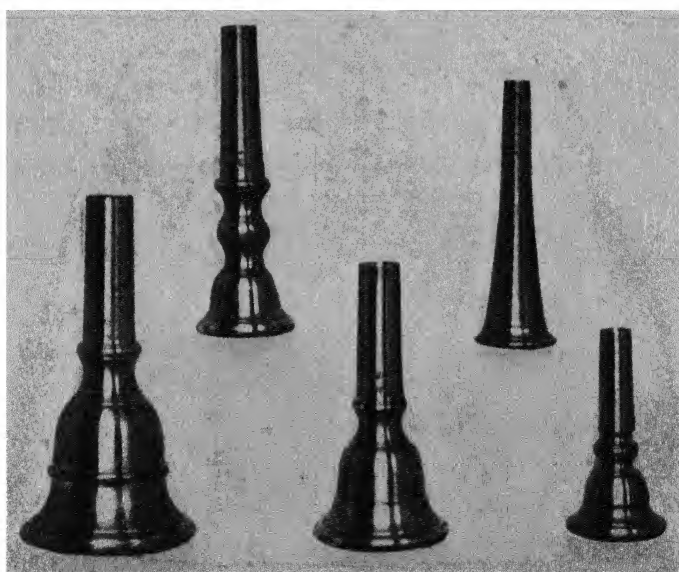FIG. 1.—MOUTHPIECE OF CLARINET.   FIG. 2.—MOUTHPIECE OF OBOE

FIG. 3.— MOUTHPIECES
Top Row. Trumpet and French Horn.
Bottom Row: Tuba, Trombone, and Cornet.   (Besson & Co.)

PLATE VIII

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p. 51) contains the whole series of harmonics, with a heavy preponderance of some of these over the fundamental (cf. p. 59), hence the "reedy" tone. Moreover, overblowing produces first the octave. In consequence, fewer note-holes are required than on the clarinet. The fundamental of the complete tube is B. By successively releasing two extension keys, and uncovering the six note-holes, we rise to *c*; re-covering the six holes and opening a vent-hole gives us *b*, and the notes of the next octave; a second vent-hole allows of overblowing to the third harmonic, i.e. the twelfth.

The analysis of the fingering is the same as that of the
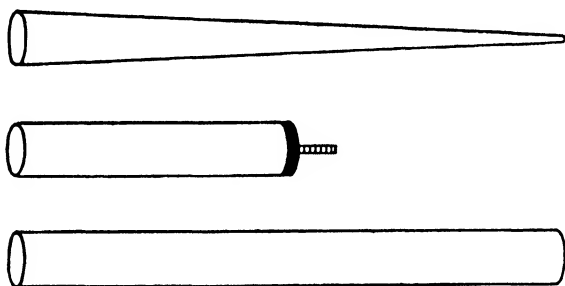
Fig. 13.—Resonance to Pipes of Various Types.

flute, i.e. that for a doubly open pipe; provided that the cross-section S, which is a gradually increasing quantity on the oboe, be measured below the note-hole in question.

The bass clarinet (being a larger edition of the clarinet) and also the cor anglais do not introduce any fresh scientific principles, but the saxophone (Plate VII, p. 55) is of interest, as being a cross-breed between the two types. The simplest way to describe the saxophone is to say that it is an oboe tube of metal provided with a clarinet reed and mouthpiece. In virtue of the conical column of air it has the full range of harmonics, and overblows to the octave—an excellent illustration of the fact that it is the

column of air which determines what harmonic is produced by overblowing, not the reed. The fingering is then the same as that of the oboe. The reed being stouter and more massive, gives the reed (and therefore the player acting on it) greater control over the vibrations of the system, but the combination does not seem altogether a happy one. Although the tone of the instrument is not unpleasant, the struggle between the reed and the air column leads to a halting speech, sometimes with introductory "buzzing" on the part of the reed. To reduce damping of the air's vibration, and so to give it greater chance to overpower

FIG. 14.—Reed Organ Pipe.

the reed, the note-holes are wide—especially on bass saxophones—and are all worked by keys, as they are too large for the fingers to cover completely.

On some of the bass wood-wind, the bassoon notably, the finger-holes are drilled obliquely through the wood to bring them within reach of the fingers. This causes the impedance of these holes to the aerial motion to be very great, thereby rendering these notes feeble and uncertain. The method used on the larger saxophones, of controlling every hole by levers, possesses the advantage that the holes can be made wide and be covered by correspondingly wide pads, so that the impedance is reduced, though at the cost of greater expense.

For comparison with the mouthpieces of these orchestral instruments, Fig. 14 shows a section of the "boot" of a reed organ pipe. The pipe and reed are tuned to the same frequency, so that the system works "at resonance," unlike the orchestral types, the tuning being accomplished by a

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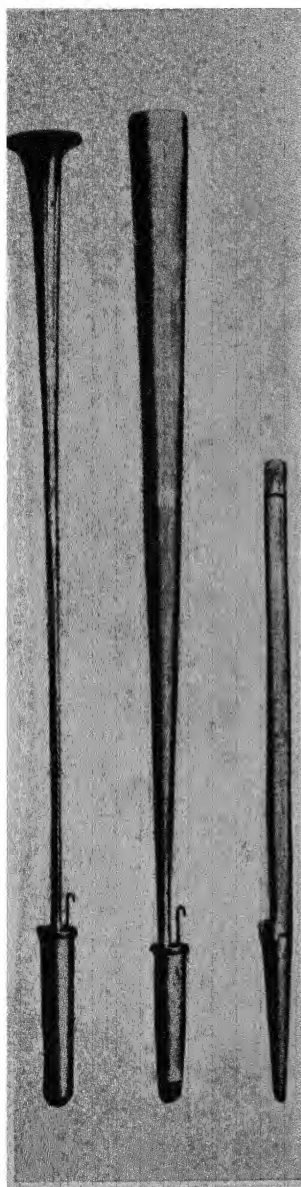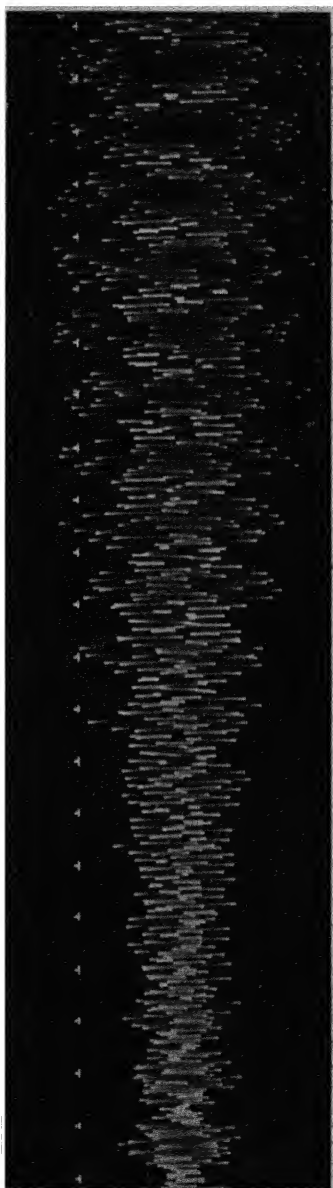
FIG. 1.—ORGAN PIPES.

TWO REEDS AND A BEARDED PIPE.

FIG. 2.—WAVE-FORM OF BELL (after Miller).

PLATE IX

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spring which presses on the reed near its clamped end, shortening or lengthening it at will. Plate IX, Fig. 1, shows two reed pipes, designed to imitate orchestral instruments, one of the oboe type (conical tube) and one with a horn.

CHAPTER IV

BRASS

I N brass instruments the column of air is excited by the player's lips extended across the mouthpiece, and set in vibration by his breath. The player's lips function like an oboe-type reed. It might be thought that there would then be no difference in the working of these systems from the reed instruments of corresponding shape, except that since the days of the extinct key-bugle, there are no note-holes for varying the effective length of the vibrating column of air. This is not so, however, as the size and shape of the mouthpiece exert an important and hitherto unexplained influence on the tone of the system. Plate VIII, Fig. 3, p. 63, shows the shapes of mouthpieces commonly employed on orchestral brass.

They vary in shape from the pure cup (hemisphere) of the trumpet to the funnel of the horn, and in size from the cornet to the tuba. The narrow end of the mouthpiece forms a tight fit into the body of the instrument, while the wide circular end fits over the centre portion of the player's lips. The player buzzes the note which he desires to produce, tightening and confining the vibrating portion of the lips for the high notes, holding the lips very loose for the bass notes, since the lips have to vibrate with the same frequency as the note of the column of air, and the higher this is, the greater stretching force and smaller vibrating mass is required of the "reeds." If the mouthpiece debouches on the tube with a sharp flange, as in the

cup form, the tone is brilliant in the sense that the higher harmonics are favoured. Playing becomes more difficult, for the adjustment of the lips and their position relative to the cup becomes more delicate, but once this adjustment is correctly made, the note comes out with great éclat, in the "blare" characteristic of such instruments. On the other hand, a tapering junction between mouthpiece and tube proper favours mellowness of tone, which though soft is produced without great expenditure of energy or finesse, but the vibration of the lips is more readily damped. We will attempt to explain the difference in terms of edge-tones.

When a jet of air issues from a circular orifice there is a tendency for it to curl up into vortex rings, though with no definite spacing between them. This is noticeable when smoke is issuing from a stack (Plate XIII, Fig. 1, p. 97). We may, however, range these vortex rings into an orderly procession by a process similar to that which we employed for the jet from a linear slit, i.e. make the issuing fluid strike an edge; but, in this case, it must be a circular edge, such as the open end of a pipe, or a hole in a plate. Under such circumstances the distance between successive vortex rings becomes equal to, or a sub-multiple of, the distance from the orifice of the jet to the circular edge facing it. The same edge-tone law relates the pitch of the note due to the vortices striking the edge with the velocity of the jet and with the distance from orifice to edge. By making this last distance very small, we can get notes very high-pitched. A familiar use of such a vortex system is made in the bird-call or decoy (Fig. 15). This consists of two thin metal plates fixed a few millimetres apart and parallel to each other. Two very small holes are drilled in these plates exactly opposite to each other. On blowing through one hole towards the other, a note is produced whose pitch

is governed by the general law of edge-tones. Now, in the trumpet mouthpiece we have such a system, but complicated by the fact that the walls of the orifice, i.e. the tissue surrounding the gap through which the breath is issuing, are themselves vibrating with the desired frequency. If that should correspond to the edge-tone produced by the vortex rings striking the flange, this tone will be assisted; and if again this corresponds to one of the notes of the trumpet column we have a tripartite system in resonance, and the note will be produced with intensity and brilliance. The
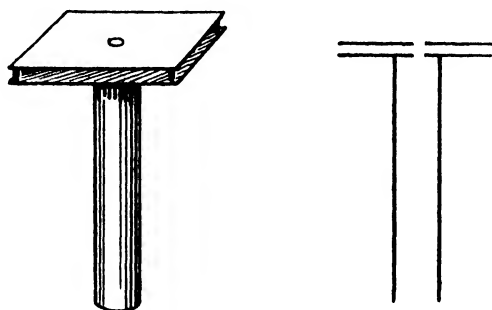
Fig. 15.—Decoy.

formula for the normal production of these edge-tones then becomes:

Pitch of edge-tone

$$= \frac{\frac{1}{2} \text{ (Velocity of issuing breath)}}{\text{Distance from lips to mouthpiece flange}}$$

So, to produce a high note, the player should, beside adjusting the tension on the lips, push the lips in towards the flange and/or increase the speed of the blast.

On mouthpieces of the horn type there is no flange and therefore nothing definite to form an edge-tone, hence the player is deprived of its help, with the result of a soft tone, not needing adjustment of the position of the lips, but

merely of the pitch of their vibration—a simpler if less effective sound-producing system.

One may remark here on the analogy of the French horn to the voice considered as a wind instrument. The vocal cords vibrate in the same fashion as the lips of the horn player; the larynx above replaces the mouthpiece tube and leads to the reinforcing air cavities of the system, the mouth, nose, etc. It appears that there is nothing corresponding to a sharp edge for the vortices, which issue from the slit between the vocal cords, to strike against. There is a possibility that the false vocal cords perform this function, though they seem to be too close above the true vocal cords to be effective in this way. So it seems that the voice possesses the flexibility and mellowness of the French horn with this added advantage, that the singer can alter the size and shape of the resonating volume of air.

In support of our assumption as to the function of the edge-tones in these mouthpieces we may cite the varying depth of the cup in instruments of different size and therefore operating in different regions of the musical scale. In accordance with our formula, the depth of the cup is greater on the bass instruments. The table on the next page shows this depth together with the breath velocity required to produce an edge-tone at the pitch of the lowest note employed on the instrument—this is not necessarily the fundamental of the column of air. This breath speed has been calculated on the assumption that the vortex rings are at a spacing equal to the depth of the cup from hole to flange, and therefore that the lips are flush with the opening of the mouthpiece. As the blowing pressure is roughly proportional to the square of the blast velocity, the last column gives the theoretical blowing pressure.

Instrument.	Cup Depth. (mm.)	Pitch.	Cup Depth × Pitch = ½ Blast Velocity. (mm. sec.)	Theoretical Blowing Pressure.
Trumpet . . .	42	174	14,600	121
Horn	44	174	15,300	123·5
Alto	49	154	15,100	122·5
Trombone . . .	64	115	14,900	122
Tuba	82	77	12,600	110

It will be observed that the depth of the mouthpieces, with the exception of the tuba, are almost in the proportion required to give the same blowing pressure and therefore same breath velocity for the lowest note on all the instruments.   This is a desideratum, as it equalizes the efforts of the players on the various instruments.   For this condition to be exactly fulfilled it is necessary that the depth of the cup should bear an inverse proportion to the frequency of the lowest note to be produced.   To aid the player in forming his lips for a low note the width of the flange of the cup should be increased for a low-pitched instrument.

It is possible to measure approximately the blowing pressure used, at the risk of some inconvenience to the player, by introducing the end of a little pressure gauge into his mouth. · This gauge consists merely of a glass tube bent into a U-shape containing a little water, the other end being open to the air.   On a given instrument the blowing pressure increases as the logarithm of the pitch, if the intensity of the note is kept constant.   This we should expect from our edge-tone formula.   On valve instruments, where it is possible to produce the same note from a longer or shorter column of air, the mode of production does not affect the blowing pressure, for the edge-tone is unchanged.

Since note-holes are not provided on the brass, the instru-

ments consisting of a simple tube ending in a bell, like the post-horn bugle and natural horn, have their musical activities confined to the fundamental and harmonics of the air column.   As the tubes are generally long and narrow, it is often impossible to produce the lowest of the series, and for musical purposes the available notes start with the octave of the fundamental.

As to the quality of the notes from brass instruments, we have already discussed most of the determining factors either in this or the preceding chapters.   Apart from the contribution of the mouthpiece, the note of the column of air is affected by:

(1) The shape of the tube, conical or cylindrical.
(2) The scale of the tube.
(3) The thickness of the brass walls.
(4) The shape of the bell.

The expansion of the tube at the bell exerts a considerable influence on the quality of the instrument.   Speaking generally, one may say that this departure from the simple cylindrical form has three effects: (1) to introduce the even partials, which, as we have seen, are absent in the simple stopped cylindrical tube—for the tube is virtually "stopped" by the lips; (2) to reduce the intensity of the higher partial tones; (3) to radiate the sound more efficiently into the atmosphere.

The tubes of most brass instruments—the trombone is a notable exception—widen out from the mouthpiece in approximately "exponential" form.   The characteristic property of an exponential section is that it expands in such a fashion that the ratio of the widths of sections at unit distance apart along the tube is a constant quantity. In Fig. 16 is shown the section of an "exponential horn" for which this ratio is equal to 17/14.   This ratio may be taken as a measure of the "flare" of the tube: we will call

it the flare coefficient. In the table are shown the approximate flare coefficients calculated from the ratio of the end areas of the tubes divided by their length for certain instruments which the author has measured. With the excep-
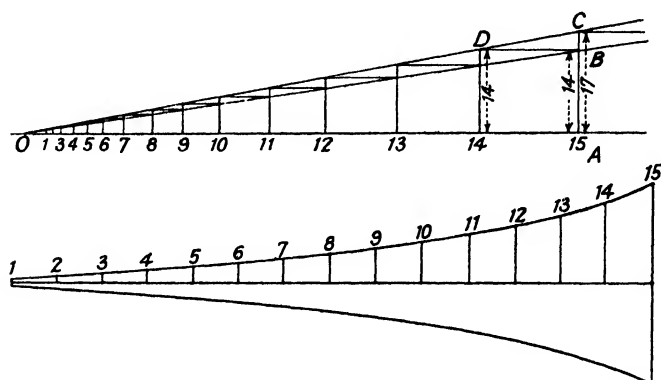

FIG. 16.—Exponential Flare (Coefficient 17/14).

tion of the French horn wherein the flare coefficient remains constant right up to the end, a more sudden expansion than these numbers indicate occurs at the final flange of the bell.

Instrument.	Flare Coefficient (between beginning of flare and bell).
Trombone	1·3
French Horn	1·25
Cornet	1·25
Trumpet	1·25
Tuba	1·1

Above the horn of Fig. 16 is shown the method of designing a horn to a given flare. On the base line at A two lines are erected, AB and AC, with lengths in the ratio of 14 : 17 (in this case). The other end of the base line O

is joined to B and C by two lines. A line is then run out horizontally from B till it cuts OC at D, where a perpendicular is dropped to the base. Proceeding in this fashion a series of steps (15 in this case) is obtained, such that the height of each is 17/14 times the preceding step. These fifteen vertical lines are now set out on another base line at *equal horizontal distances apart*, which gives the curve of the horn.

The formula which gives the resonant frequencies of the column of air in an exponential horn is the same as for an open pipe, plus a correction depending on the flare. This correction is the same for all frequencies, so that its relative effect on the higher overtones will be small, and as the player rarely uses the two lowest notes of the harmonic series available from the horn, this defect of intonation is not marked.

The exponential horn has the advantage that the gradually increasing curvature forced upon the sound waves which leave the mouthpiece and pass down the tube transforms them more readily into spheres diverging from a point just beyond the open end, whereas the waves passing down a cone without a bell have their curvature suddenly changed at the open end. In consequence of the less abrupt "turn-over" to the atmosphere of the curves in the first case, the flaring horn sends out sound more efficiently into the auditorium. Of the two reed pipes in Plate IX, Fig. 1, p. 67, apart from difference of timbre, the "straight cone" will be less efficient as a sound-transmitter.

By using the "exponential formula" an instrument-designer can calculate the best flare for a tube of given length.[1] In actual manufacture the tube is made conical for the early part of its length and then widens out, but the

[1] For the mathematical theory of the exponential horn, see Crandall: *Vibrating Systems and Sound*, pp. 157-74.

calculation still holds within the limits of construction. If you give a tube a large flare you reduce the intensity of the upper partials in the note and render it more mellow. Compare the French horn with the cornet. Fig. 17 shows the rapid drop in the height of the resonance peaks of a horn of moderate flare. It is well known that exponential horns of considerable flare are used on gramophones and loud-speakers. The function of these is just opposite to
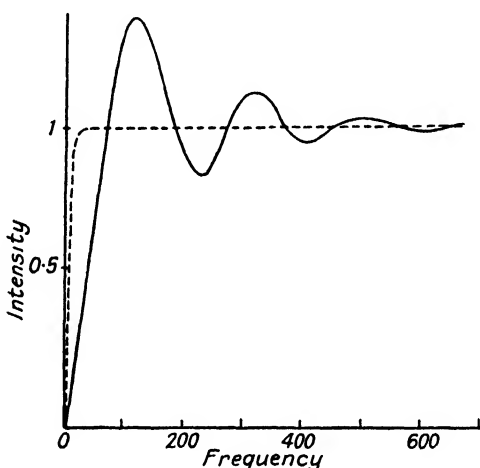

FIG. 17.—Resonance of Exponential Horn.

that of a brass instrument. The designer of the latter wants very definite resonance at certain frequencies; the man who designs a gramophone horn wants equal responses at all audible frequencies corresponding to the dotted line on the figure, and no marked resonances with annoying distortion of the music; that is why he employs a very flaring horn which will respond uniformly to the whole gamut. This fact should warn the designer of brass instruments against employing bells of large flare, lest by so doing he renders the tone dull and the natural frequencies of the instrument ineffective and difficult to pitch on. If

FRENCH HORN.
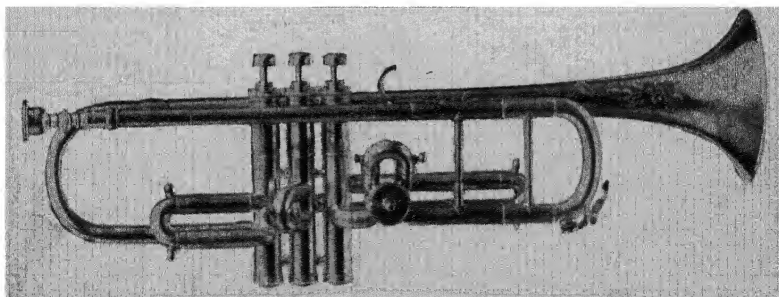
TRUMPET, with Rotary Valve to change Crooks.

Plate X (Hawkes & Son).

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in doubt, give a player a loud-speaker horn for a brass instrument!

In considering the characteristics of the individual brass instruments we will commence with the French horn (Plate X). We have already traced its peculiar timbre to the unbroken shape of its mouthpiece and the rapid widening out of the bore and bell in relation to the length of the tube. Owing to the non-resonant form of the mouthpiece, the player has a greater facility to correct, by means of his lips, any fault of intonation in the notes of his instrument; but he lacks the intensity obtainable from other members of the brass family. Nevertheless, the player on the natural horn in FF could produce only the harmonic scale (minus the fundamental) from his instrument, by the aid of the lips alone. The fundamental of the instrument could be lowered a semitone by introducing the hand partially into the bell, and a new series of harmonics based on EE were available. The old horn-player had still another dodge for use: this was to introduce the hand, fingers foremost, completely into the bell to produce certain extra notes. This had practically the same effect as the modern pear-shaped mute.

Prof. Kirby notes the possibility of producing chords on the horn by setting the pistons and lips to give one note and humming another, but it is doubtful whether this can be done when the two notes do not both form part of the harmonic series of the instrument, that is to say, the humming probably serves to emphasize one partial tone which is already present in the column of air in the instrument.

But the apparatus which gives to all modern orchestral brass, with the exception of the trombone, its characteristic appearance, is the pistons and crooks. These embellishments were introduced in order to give these instruments

equal facilities with the wood-wind for playing modulated music, by providing a complete chromatic scale. Obviously, if we can open out the tube at some portion of its length and introduce an additional length of tubing—called from its shape a "crook"—we lower the fundamental of the column and have an instrument which gives the harmonic series of a new fundamental. We increase materially the number of notes playable on the instrument if this added crook, instead of being an occasional addition to the instrument, is permanently fixed thereto, but normally closed by a valve or piston, except when the player desires to use one of the notes of the new harmonic series. It will readily be perceived that, with several such crooks available, the gaps in the scale can be bridged—nay, more than bridged, for some notes will be obtained on more than one crook. Seeing that more than one crook can be added to the tube at the same time by depressing the appropriate pistons, three valves are found to be sufficient on the horn to cover the chromatic scale from F upwards.

The use of two or more pistons leads to faulty intonation unless a compensating device is employed. Thus, if the first piston alone introduces a crook which lowers the fundamental by a semitone, while the second piston alone lowers it by a whole tone, the two used together might be expected to lower the pitch by three semitones, and at first sight it seems that this desideratum is effected. But the first increases the speaking length in the ratio 15 : 16, and the second in the ratio 8 : 9, and hence the two together lengthen it by $\frac{1}{15} + \frac{1}{8} = \frac{23}{120}$, which is less than the increase necessary, $\frac{24}{120}$, for three semitones. To overcome this fault, short compensating tubes are added which come automatically into action only when the appropriate combination of pistons is pressed, and which bring the total added length to the amount necessary (Mr. Blaikley's system).

The three pistons on the horn actually produce lowerings of one, two and three semitones respectively when used separately.

It is to be noticed that these crooks, being of cylindrical bore, introduce a change of form into the middle of the tube, and to this is ascribed the somewhat trumpety tone of the valve-horn as compared to the old natural horn. On one system (Rudall Carte) this change of form is obviated.

The trumpet (Plate X) is cylindrical in form for about three-quarters of its length, it then widens conically, ending in a moderate flare at the bell. Owing to the very definite cupping of the mouthpiece, the player must get resonance between the edge-tones there produced and the column of air, in order to make the instrument speak effectively, and cannot coax it into or out of tune by his lips. The noble retinue of harmonics to which the characteristic blare is due must be ascribed to the narrow bore of the tube and moderate expansion in the bell. The cornet is a smaller edition, having a wider tube and flange, and consequently less brilliant tone. Both are provided with three valves, and besides these there are other loose crooks which may be added if time is allowed in the performance for their affixing, which produce more permanent changes of the fundamental notes of the instrument. Latterly these changes have been made automatic: cf. the rotary valve on Plate X. As the cornet is a transposing instrument, these added crooks do not affect the player's fingering. The instrument is set to produce the written note, but the actual note depends on the crook which has been added; the actual note answering to the composer's intention. Crooks of considerable length are not to be recommended, for the pistons and compensating pistons are designed for a given total length of tube, and if this length be changed the valve-tubes will be in faulty intonation among themselves. A

mute of pear-shaped metal may be added which practically makes the instrument a cylindrical pipe, stopped at both ends. A stifled effect is produced if this is blown gently, but blown fortissimo a quite new metallic timbre is produced in which the higher overtones predominate, and which is apparently an essential feature of Eastern slave-markets. A mute on an instrument of conical bore produces an even more raucous noise when overblown, for the tones of a cone closed at the wide end form an inharmonic series in the ratio: 1·43, 2·46, 3·47, 4·47, 5·48, etc.

The tubas and saxhorns are constructed on the same principle. They are another hybrid invention of Adolphe Sax. The tuba is a cone of comparatively wide angle like that of a horn, but a cup-shaped mouthpiece is fitted. On the true tuba, of which the euphonium is an example, the tube is very long and expands to a foot or more in diameter at the bell flange. Owing to the fact that the average width of the bore is so large and that the cup of the mouthpiece is so deep (cf. p. 73), it is possible in these instruments to produce the fundamental, which on the bass-tuba is as low as FF; or even a few forced notes *below* this, by employing a very loose lip. To fill the gaps chromatically four valves are necessary. These instruments are much employed to fill in the bass of military and other outdoor bands on account of the power which such a large vibrating mass of air as they contain is able to put into these low notes.

The trombone (Plate VII, p. 55) differs from all other wind instruments in respect that there are no fixed lengths to the column of air producing the notes. By the use of the "slide"—a long U-shaped crook which can be telescoped over the mouthpiece end and bell end of the tube, without allowing of pressure leakages—it is possible to alter the length of the tube continuously, instead of step by step. No valves are necessary, as the player can readily

cover the chromatic scale by employing the harmonic series
of a tube of any length he chooses between the limits of the
stretch of the slide.   Eight positions of this infinite variety
of tube-lengths are found sufficient in practice, but these
positions need not become stereotyped, for the trombone
shares this advantage with the strings, that the player can
make his own notes, adjusting the position of the slide to
get just intonation.   By continuing to blow while moving
the slide he can pass continuously through a series of notes
in that *portamento* which is only too common at the present
time in trivial music.

The bore of the trombone is cylindrical except at the
end where it widens out into the bell, and therefore its
tone partakes of the character of the trumpet, but with
the even harmonics less marked than the uneven ones.   It
would not be possible, of course, to apply the slide to any
but an instrument of constant bore throughout the great
part of its length.

The slide form of brass instrument involves fewer con-
volutions of the tube than those types in which the sound
has to traverse a tortuous path through a number of valves
and crooks.   Although it is sometimes thought that the
sound is guided round these bends like a flow of gas along
a conduit, there is evidence to show that the sound waves
proceed by a series of reflections off the walls of the tube
round such a bend, with a little loss of energy at each
reflection.   Although this loss is small, in virtue of the
fact that the metal is so rigid, yet the straight tube of the
old post-horn, now enjoying a resuscitation in the "straight"
trumpet, was, in this respect, a better form.   This latter
type of trumpet—sometimes called the Bach trumpet—
has, it is true, its unswerving length broken by the insertion
of the three pistons with their accompanying "bends," but it
seems to possess a brilliance not to be attained by the more

familiar curled type. It is tuned by inserting an additional straight joint of tube near the mouthpiece, in place of a crook.

In Plate XI are shown reproductions of Dr. Mary Browning's records of a number of sound changes by a professional player on the cornet and trumpet. The first shows a series of detached notes going up the scale marked in Tonic Sol-fa notation on the record. The rise in pitch is well shown by the increasing closeness of the vibrations. Notice that the vibration never dies down entirely between the separate notes although the scale was not slurred, and the different times spent by the player on individual notes. The second shows a rapid reiteration of the same note by striking the lips with the tongue. The triple tonguing used here is shown by the triple swelling out of the record. The third record is of a slurred jump from c_1 to f; the time taken from x where the change begins to y where the new note settles down is about one-tenth of a second, and is filled with irregular "beating" between the old and new notes. Number four is a "turn" round a main note of frequency 538, the turn starting at z; the alternation between shorter and longer waves is seen to be very rapid. The alternation of successive notes is still more obvious in the last pair of records, which are shakes on the C trumpet, firstly between $f\sharp$ and g—the g is an "open" note, and the $f\sharp$ is produced by depressing the second valve— and underneath is a shake on the low B. In both of these the duration of the pair of notes is unequal, and in the former the open g persists throughout, giving throbs with the $f\sharp$. The pair of notes—one unit of the pattern—take about one-eighth of a second. The general conclusion drawn by Dr. Browning is that the ear is less sensitive to the changes of sound than is the apparatus used as detector, and she remarks: "It is well for the acceptance of the current standard of playing that the ear is not more keenly critical."
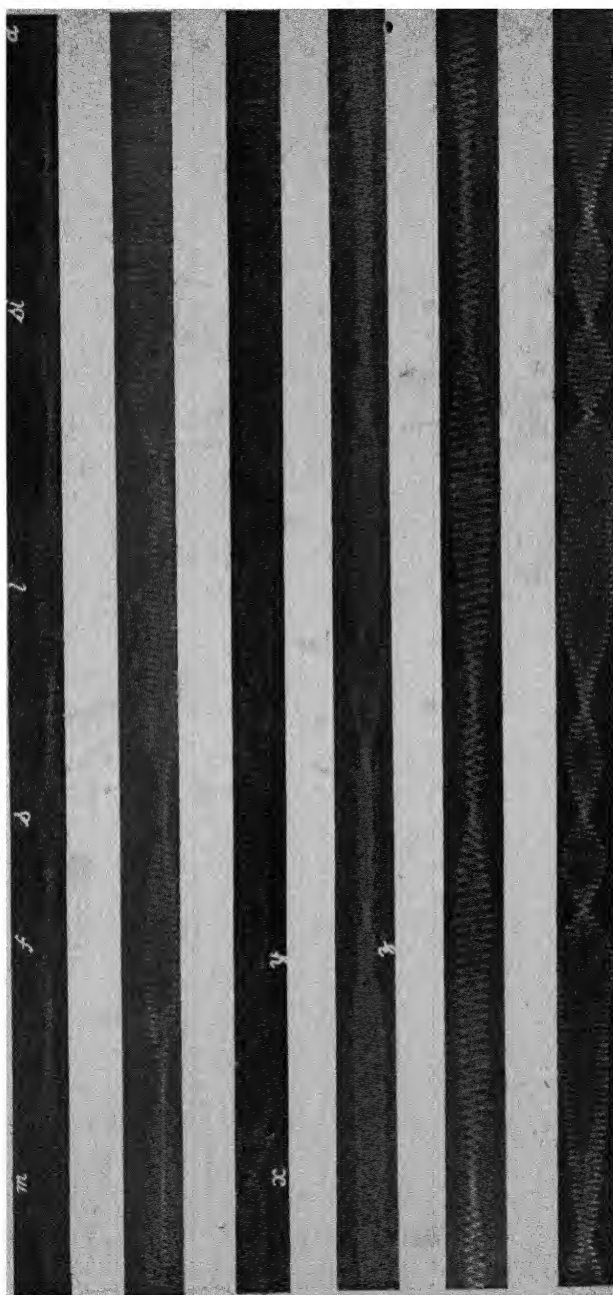
CORNET AND TRUMPET RECORDS (after Dr. Mary Browning).

PLATE XI

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CHAPTER V

PERCUSSION

THE instruments whose vibrations are excited by a blow may be divided into three classes, according to the vibrating system used: as bars, plates or membranes, and bells. The pianoforte family ought to come under this classification, but they are usually classed along with the bow instruments.

When the vibrating material is a bar or a strip of wood or metal, it may be either clamped tightly at one end, leaving the other free to vibrate, or it may be lightly supported at two points. The reeds of the wood-wind and of the harmonium are fixed in the former fashion. Such bars are not used alone in the orchestra, as their output of energy is weak and the noise they produce rather discordant. This is because a bar or reed clamped at one end does not give the natural series of harmonics, but a series of tones the ratio of whose frequencies is much more complex. When the vibrating reed is coupled to a resonator or other cavity of air such as that in the clarinet, the tones of the cavity are harmonic and therefore smother the inharmonic overtones of the reed. This principle is used partly with the object of cutting out these discordant overtones, and partly to reinforce the fundamental of the bar, in certain keyboard instruments containing graduated bars fixed over the mouths of tuned resonators (*celesta*) or tuning forks whose stems are stepped into "resonance boxes." Plate XII, Fig. 2, shows a series of such forks with resonance boxes

containing cavities of air tuned to the fundamentals of the forks. The cavity, of course, decreases in volume with the size of the fork, i.e. as the pitch goes up. The factors on which the pitch of such bars or reeds depend are the elasticity and thickness directly, and the length inversely.

The xylophone is the only orchestral example of the "supported bar" instrument. The bars are of hard wood pierced at two points about a sixth of the length from each end and held at these points by loosely fitting screws in a rack. These points are chosen as being nodes of the bar when it is vibrating transversely with antinodes at the
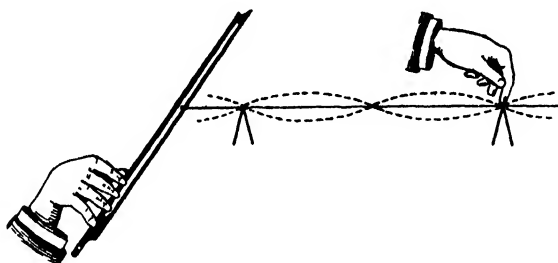

Fig. 18.—Mode of Vibration of Supported Bar.

ends and intermediately. We can show the correct position for the nodal supports by laying a bar of metal sprinkled with sand on two little prisms, and bowing or striking one end with a rubber hammer, while the finger keeps it down on its supports. When the correct position for the supports have been found, the sand will be thrown off the centre segments of the bar (Fig. 18).

The pure tone of the xylophone, which is well known if only from Saint-Saëns' use of it in the famous Danse Macabre, is due to the fact that the overtones have nodes at other points and so are not excited. Resonators are sometimes added, but these are only reinforcers.

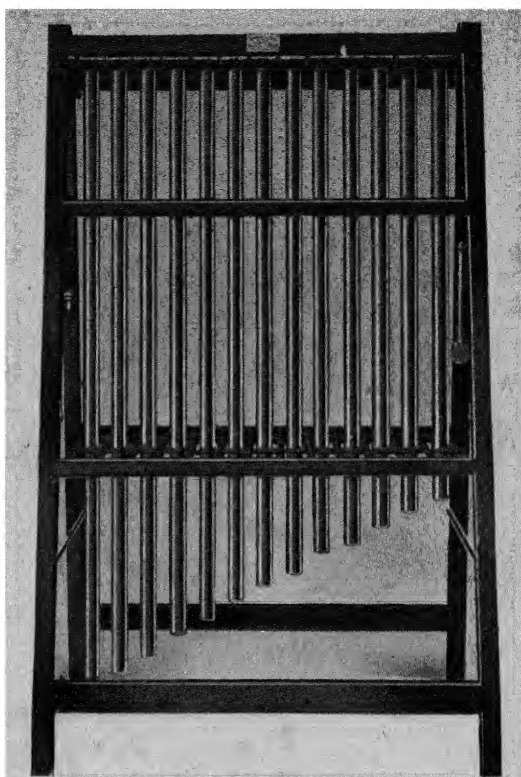The note of the triangle—a steel rod bent to this shape,

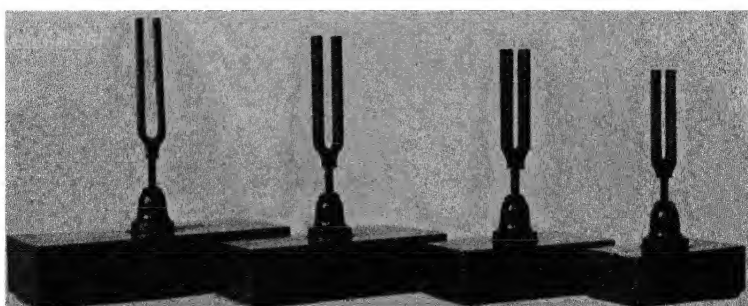FIG. 1.—TUBULAR BELLS Hawkes & Son).

FIG. 2.—SET OF TUNING FORKS ON RESONANCE BOXES.

PLATE XII

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hung from one end—contains many neighbouring over-tones forming an indefinite noise, and, as a rhythm marker, is not tuned to a definite pitch.

The drums employ a vibrating membrane stretched over an air cavity of cylindrical or hemispherical shape. The possible modes of vibration of a drum-skin are governed by the stationary nodes which may form on the membrane. These nodal lines are either diameters of the membrane or smaller circles within its circumference. We can demonstrate their existence by Chladni's device of strewing sand over the vibrating surface. The sand collects in the places where there is no up-and-down motion of the skin. Since the membrane at its circumference is fixed to the drum, this circle is always a node.

The simplest mode of vibration is, then, that in which the whole surface of the skin moves up and down within the limits of the restraint imposed by the attachment. This gives the fundamental tone. The next is due to a single nodal diameter; in this case, while the left-hand section moves down the right-hand is moving up, and, to a certain extent, the two halves "interfere" with each other's influence in the surrounding air. Proceeding in order of pitch, we have two nodal lines, then one nodal circle, etc. These nodal lines and relative pitch to the fundamental are shown in Fig. 19.

It must be understood that when the skin is struck all these tones generally combine to form the characteristic, rapidly damped, noise of the drum, and it will be observed from the irregularity of the pitch ratios that this noise is inharmonic. Thus the sixth of the series is a little less (2·92 instead of 3) than three octaves above the fundamental. Apart from the fact that a padded stick gives prominence to the lower tones, some of the more objectionable overtones can be removed by employing a principle common to all

instruments having membranes or strings, i.e. that those overtones which would have a node at the point of excitation are silent. This is not strange considering that the point in question is most violently agitated, whereas such an overtone requires it to be still. Thus by striking the drum at half the radius from the edge, as do most players, the tone with one nodal circle (frequency 2·3 times the fundamental) as well as higher ones in the series are cut out. The technique of drumming consists mainly in a quick
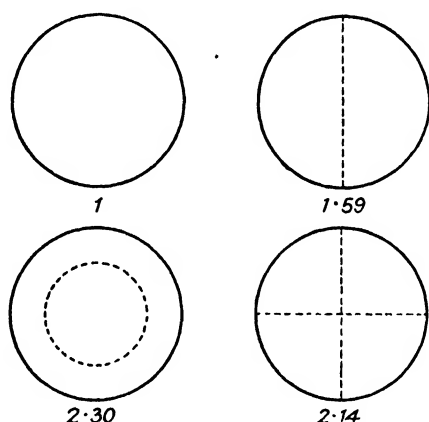

FIG. 19.—Modes of Vibration of Drum-skins.

removal of the stick to prevent the vibrations being rapidly damped.

The fundamental of the drum-skin is inversely proportional to the diameter of the skin, and directly as the square root of the tension in it, which alone can be altered. It also depends on the material of the skin, varying in fact inversely as the square root of the weight of unit superficial area of the skin. This is, of course, for a skin of uniform texture; but it has been proved theoretically that if the weight or thickness of the skin increased progressively from the rim to the centre, the tones would be harmonic to one

another.   This is interesting as Prof. Raman has noticed certain Indian drums in which the skin is loaded beneath by a bag of iron filings getting deeper towards the centre. Some such expedient might be employed in orchestral drums to purify the tone, though it must be confessed that such loading would act as a rapid damper upon the vibrations.

It might be thought that the inharmonic series of tones proper to a membrane would preclude·its employment as a musical instrument.   As a matter of fact, most drums are mere rhythm markers, and the inharmonic complex note which they give is not meant to form part of the harmony. Exceptionally the kettle-drums (*timpani*) are tuned to a definite pitch.   These have skins stretched over cavities of air in shell-shaped cases, the tension in the membrane being adjustable by turning-screws.   The size of the air cavity *ought* to be chosen to give the same fundamental pitch as the skin, in order that, in the coupled-system membrane $+$ cavity, the inharmonics of the former might be eliminated for the same reason as those of a reed in a reed instrument.   This would require one drum to every note required by the composer, but usually the latter limits himself to write for two or three drums at a time, directing the player to retune the skin when a new key is required, so that resonance between the skin and the cavity of air cannot be secured.

The military drum is interesting for the fact that it employs one skin at each end of a cylindrical drum, the lower one being crossed along a diameter by a stretched piece of gut or steel wire called a snare.   The upper skin is played upon by the drum-sticks, the lower then vibrates in sympathy, and in doing so beats upon the snare, producing a peculiar rattling timbre.

In the orchestra the nearest approach to the vibrating

metal plate is afforded by the cymbals. Theoretically, a membrane possesses no elasticity of its own, it vibrates only when stretched. A plate possesses sufficient stiffness to vibrate without this aid, as, for instance, when it is dropped on the floor. Alternatively a plate may be regarded as a very wide metallic reed.

The cymbal consists of a circular plate sunk at its centre, where it is supported by the handle. Consequently, the centre is always a node. Substituting a plane circular plate, the possible modes of vibration, with their corresponding nodal lines (diameters or circles), can be shown by strewing fine sand and tapping with a light hammer. Photographs of some of these sand patterns corresponding to higher members of the inharmonic series of overtones are shown in Plate XIII, Fig. 2.

Very often when the cymbal is struck with a drum-stick a waviness in the tone may be heard: this is due to the fact that the nodal lines tend to rotate, causing sections in opposite phases to pass in succession before the listener, thus giving a slow rise and fall in intensity.

Bells are often employed in the percussion department. Their genesis is the bent plate, and they may be regarded as an extension of the cymbal. From the physical point of view the half-shell of uniform thickness is the simplest type. Chinese bells are of this type, and an ordinary finger-bowl held by the stem and struck on the rim gives a good imitation of their sound. The nodal lines—in the simplest mode there are four—are lines of longitude to the hemisphere. We can demonstrate their existence in the finger-bowl after bowing it, by a pith ball suspended so as to touch each point of the rim in turn (Fig. 20). The point struck naturally becomes an antinode, and when the ball touches it and the other three "points of the compass" it is violently thrown off. We often find a slow rotation

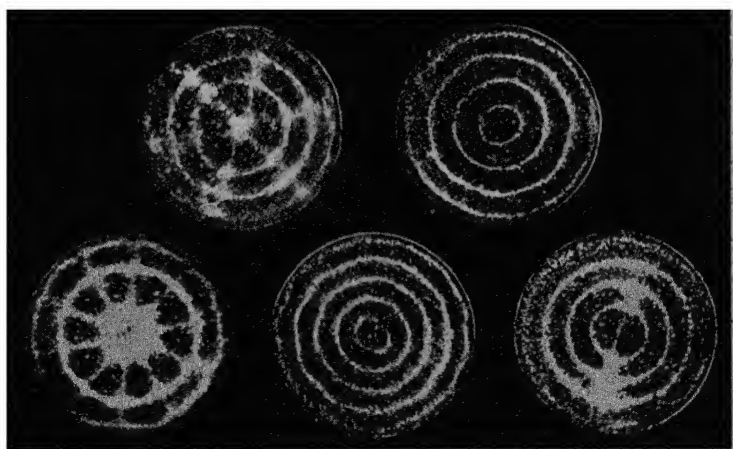Fig. 1.—EDDIES IN STREAM ISSUING FROM A RING-
SHAPED ORIFICE.

Fig. 2.—SAND FIGURES ON VIBRATING PLATE (after Schulze).

PLATE XIII

of the nodal lines round the surface of the bell corresponding to that observed on the cymbals, causing to a stationary hearer a slow waxing and waning of the sound.

The tinkle characteristic of bells of the above type is due to the predominance of the higher-pitched inharmonic forms of vibration over the fundamental, and it has caused their relegation to the "kitchen" along with the cymbals and triangle. In order to get a deeper and less discordant ring from a bell two forms are in common use.

In that form which, from long association, is known as the church bell, the energy is kept in the lower partials by
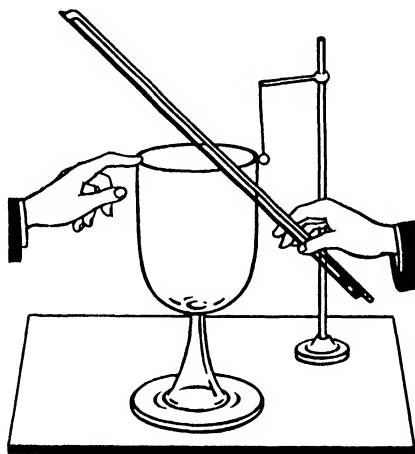

FIG. 20.—Bell-like Vibrations of a Glass Bowl.

thickening the rim at the level where the bell is struck. The bell hangs down with the rim lowest, the section of the metal thickening to just above the rim and then gradually diminishing to the rim itself. The "clapper" hangs loosely inside, and when the bell is "swung" strikes the metal at its thickest circumference, called the "sound-bow."

The fundamental pitch of the bell depends on the internal

diameter; the greater this is, the lower the note. This relation between pitch and size may be observed on the photograph of the Loughborough Carillon, Plate XIV. When being tuned the bell is placed on its crown, while a revolving table above supports a cutting tool which turns off shavings from the inside of it. To allow of such tuning, the bell is made a little too large in the mould.

The results of analyses of a large number of bells show that the various partial tones formed by division into a number of segments form a series as follows:

Approximate Relative Pitch.	Nodal Lines.
1. Fundamental	Four sectors.
2. Octave	Four sectors and ring.
3. Octave and minor third .	Six sectors.
4. Twelfth	Six sectors and ring.
5. Double octave	Eight sectors, etc.

It is to be understood that these relations are approximate only, as the tones of a bent plate are inharmonic, but the bell-founder, by adjusting the thickness at various sections, strives to make the lower tones as nearly harmonic as possible. In practice only one nodal ring is formed on a bell: this eliminates some of the discordant overtones pertaining to a plate. Besides these tones which can be approximately predicted by applying the theory of plate vibrations, there is another tone which, immediately after striking, overpowers these, but decays more rapidly. This is known as the "striking note," and its pitch, by which the founder names the bell, seems to lie near that of the octave or second in the series of partial tones; in a good bell the striking note is made coincident with or harmonic to this second overtone, even if the other overtones have to
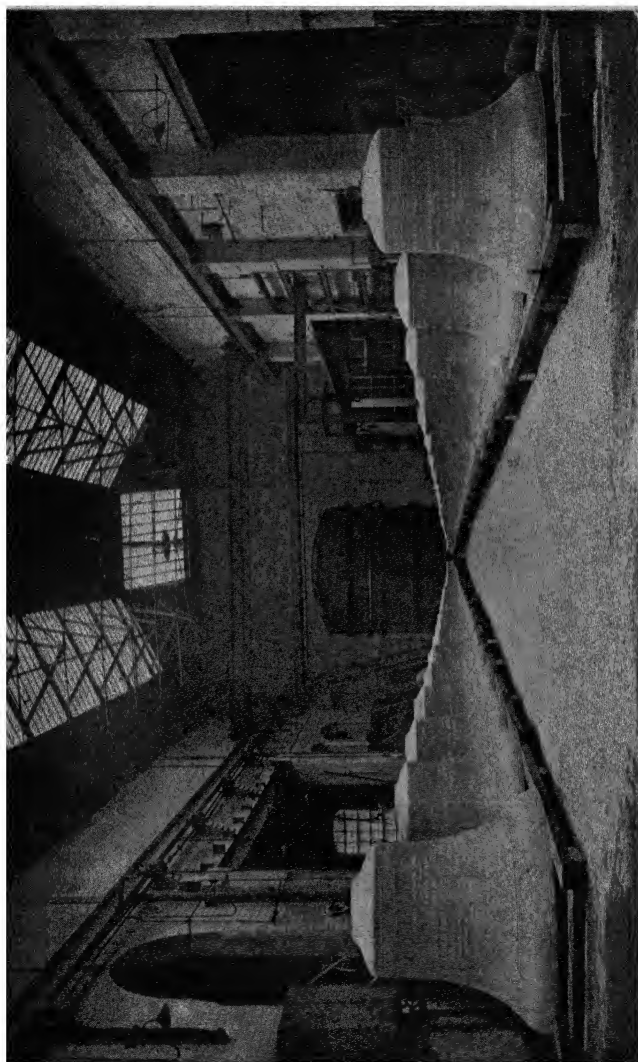
THE LOUGHBOROUGH CARILLON (Taylor & Co.).

PLATE XIV

be left inharmonic to each other. The occurrence of this striking tone is very curious, and has so far baffled explanation. The partial tones can be elicited by resonance with a tuning fork, but not so the striking note; nor can the latter be picked up by a resonator, or made to produce beats with a neighbouring tone.

Herr Griesbacher, who has tested some hundreds of bells, finds that he can make a bell resound to a fork of the striking-note pitch provided that the stem of the vibrating fork be *pressed* on to the rim. He seems to think that it is due to compressional rather than to flexural waves in the metal. The difference may be illustrated by means of a metal bar. If the bar be clamped at one end and struck a sideways blow at the other, it executes lateral vibrations of a low pitch. If, however, it be held in the hand and struck with a hammer on the end and in the direction of its length, a wave of compression will pass along the bar, giving rise to a high-pitched note.

On the other hand, Prof. Taber Jones, who has studied American carillons—founded in England, by the way—rejects this theory, as the dimensions of a bell do not give the correct pitch for compressional waves. As the striking note is more prominent when the lower tones are loudly elicited—indeed, is absent in a "tinkling" bell—he suggests that it is a subjective effect, that the peculiar mechanism of the ear gives the impression of this tone to the brain when it is jarred by the loud clang of a large church bell. Such subjective tones, having no physical existence outside the ear, and being therefore not detectable by resonators, are not unknown in other branches of sound. Prof. Taber Jones has carefully examined the pros and cons of this problem and concludes that the striking note is really the double octave (number 5 in the above series), but that its true location is masked by the general noise

of the bell, and the ear misjudges it to be in the lower octave.

Plate IX, Fig. 2 (p. 67), shows the very complex waveform, and Fig. 21 shows the relative intensities of the different overtones of a church bell.

The fifth partial is certainly very prominent just after the bell has been struck (cf. Fig. 21*a*). This then rapidly dies out, leaving the third partial master of the field a few seconds later (Fig. 21*b*).

Needless to say, as in all instruments of percussion, the material of the bell plays an important rôle. Elasticity of
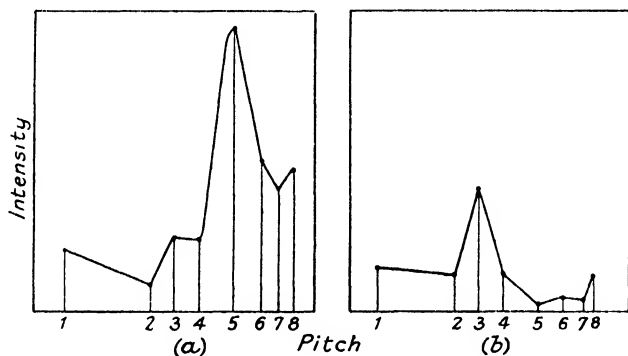

FIG. 21.—Intensities of Partial Tones of a Church Bell: (*a*) Immediately after striking ; (*b*) 3 secs. after striking (after Taber Jones).

the metal and good hanging are essential for sustained vibration. A special alloy (bell metal; 3 to 4 parts copper and 1 part tin) has been evolved to meet that requirement. Silver and gold, though more expensive, are not superior to this alloy for bell manufacture, though steel has been used as a substitute.

To save space in the orchestra and now in some organs, the percussion department employs suspended tubes to imitate the bell timbre. A number of metal tubes are supported on a frame by strings passing through a point

near the upper end, and are struck by a hammer just below the point of suspension (Plate XII, Fig. 1, p. 91). The pitch depends on the length and thickness of the metal, but the quality is complicated by the contained column of air. As in the kettle-drum, it would be an advantage if the maker chose the size and metal so that the column of air inside (regarded as an open pipe) resounded to the fundamental of the "bell" note, and so reinforced this.

CHAPTER VI

STRINGS

THE vibrations of a cord or wire stretched between two pegs is one of the oldest ways of producing a musical note, and probably dates back to the invention of the bow and arrow. Instruments formed of a number of strings stretched over a frame formed part of the "orchestras" of the ancients and all primitive peoples. The ancient Greek philosophers investigated in a crude fashion the relation between the length and tension of a stretched string and the pitch of the note given out by plucking it, but it was left to the Abbé Mersenne in the seventeenth century to give these laws a rigorous formulation. In an interesting treatise published by him in 1636, entitled *Harmonie Universelle*, he established that the pitch of a note from a stretched string—

(1) is inversely proportional to its length;

(2) is directly as the square root of the stretching force: as he says, if two strings of equal length and thickness are stretched, one requires a force four times that on the other to produce the octave of the note of the other;

(3) is inversely as the diameter of the wire, in wires *made of the same material.*

Thus, low pitch is favoured by length, thickness, and slackness of the wire, and conversely.

When wires of different material are used, it is found that the density is of import. The third relation is more fully

expressed by saying that the pitch is inversely proportional to the square root of the mass of material in unit length of the wire.

Combining the second and third relations we find that the pitch is proportional to—

$$\sqrt{\frac{\text{Stretching force}}{\text{Mass per unit length}}},$$

this quantity being in fact the velocity with which the waves run up and down the string.

Now when the string is displaced at any point waves travel to and fro along the string, being reflected at each end just as at the stopped end of a pipe, only with this difference, that the displacement—the hump on the string —is *perpendicular* to the direction in which the waves travel, instead of being *in* this direction of travel.

This being so, we have only to determine the distance between successive nodes, multiply this distance by 2, just as in the pipe, and we have the wave-length of the note in the string when so vibrating. Since the velocity of the waves is the product of the pitch or frequency and the wave-length, we must divide the expression for the velocity above by this wave-length, to get the pitch. Since the ends of the string are fixed, these must be nodes always. The fundamental vibration is that shown in Fig. 22*a*—in which the string vibrates to and fro in one loop, from the position shown by the continuous line to that of the dotted line; in this case the wave-length is twice the length of the string, and the fundamental tone is

$$\frac{1}{2 \times \text{length}} \sqrt{\frac{\text{Stretching force}}{\text{Mass of unit length}}}.$$

Fig. 22*b* shows the next possible form, with an additional node at the centre which may be encouraged by lightly

touching the string at its mid point so as to keep that point still—this being the method adopted by the violinist or harpist to produce an harmonic.

The wave-length is now equal to the length of the string —half of what it was—and therefore the pitch is doubled, since the velocity of the waves is unchanged. Thus, touching the centre produces the octave. Fig. 22c shows the next harmonic, three loops produced by touching at one-third of the length; the pitch is now three times that of the fundamental, and so on.
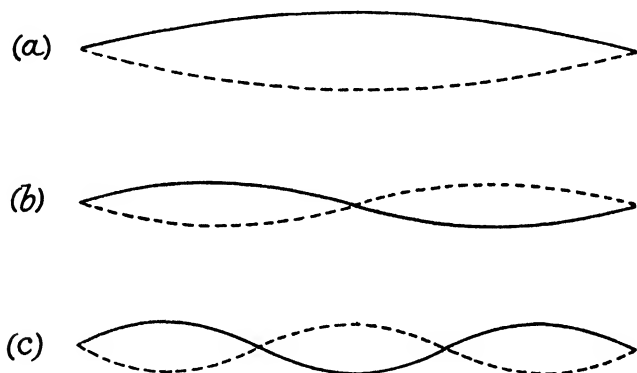
(a)

(b)

(c)

Fig. 22.—Vibrations of Strings.

The stretched string therefore is capable of giving the full range of harmonics with frequencies in the ratio 1; 2; 3; 4; 5, etc., like the *open* pipe.

In general, it produces a complex tone containing a mixture of these, of diminishing proportions as the harmonic series is ascended. The number and magnitude of these depend upon how and where the string is excited, but are governed by an important though fairly obvious provision known as Young's Law, i.e. that a node cannot be formed at the point of excitation, and therefore all partial tones (harmonics) which would have nodes at this

point are missing from the complex note produced. This law determines at what point the string shall be displaced, as we shall point out later.

To the extent that the pegs and bridges supporting the string are unyielding, there is no "end-correction" to the wave-length of the sound-waves corresponding to that of the open pipe. If the supports yield so that they are pulled to and fro with the string, the two arcs into which the latter is necessarily bent will obviously meet at some point beyond the end, and an end-correction will be required. On the other hand, if the string is very thick and stiff it will not be able to bend sharply at the bridges and the arcs will meet, and the nodes will lie, at two points a little short of
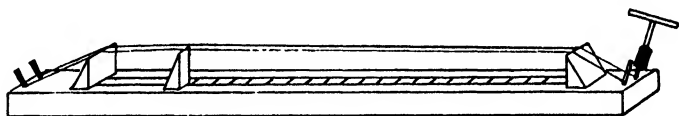

FIG. 23.—Sonometer.

the ends. This negative correction becomes of importance in the thick wires forming the bass of the piano. With thin wires and rigid end-supports, the relationship between pitch and length is so sure that it may be used for a simple and rapid comparison of pitch. It is only necessary to have a steel wire stretched over a board or sound-box between two bridges, with a loose bridge which can be moved under the wire between the fixed ones. Such an instrument is called a monochord or sonometer (Fig. 23).

The loose bridge is shifted until one segment of the wire is in tune with the pitch of a standard tuning fork, and the length of the segment measured. On repeating the adjustment to a note of different pitch the two lengths

are inversely proportional to the respective pitches of the notes, so that the unknown is found in terms of the standard.

Apart from the obsolete method of exciting strings by blowing wind across them, as in the Æolian harp, there are three ways in which the vibration can be started:

(1) by plucking the string;
(2) by striking it with a hammer;
(3) by bowing it.

The first two methods have similar results as far as the tone goes. The string being displaced and let go, the sound continues with diminishing intensity, only as long as the vibrations are not damped out by the resistance of the air and internal friction in the string; whereas a bow maintains the vibrations.

Apart from the violin type of instrument played *pizzicato*, the only instrument of the first class employed in the symphony orchestra is the harp, in which the strings are plucked by the fingers. Nevertheless, we could not completely illustrate the acoustics of plucked-string instruments without referring to the banjo type in which the strings are plucked with a sharp piece of horn called the plectrum. Every one must have noticed that the tone of the banjo is much more metallic than that of the harp. The fact is that on all stringed instruments the number and relative magnitude of the harmonics in the note are largely determined by the initial form into which the string is bent before being released.

If the string could be bent into a bow shape like that of Fig. 22*a*, it would pass to and fro between the two positions marked on this figure and give a pure tone of the fundamental only. If, however, it be bent into a "kinked" form by plucking with a hard pointed substance near the end, it goes through the series of evolutions shown in Fig. 24*b*. Two corners run from the plucked point A

in opposite directions along the string (Fig. 24a), are reflected from the end, and finally meet at the opposite point (A′ in No. 6); return is made to No. 1 position by a reverse series. The evolutions, of which six stages are here shown, can actually be observed by means of a stroboscope or slow-motion camera. Such a kinked curve when analysed is found to contain a large number of harmonics of diminishing intensity, hence the "metallic" timbre.
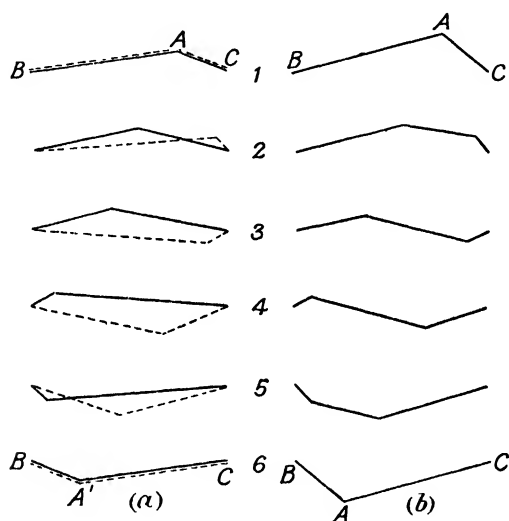

FIG. 24.—Displacement Curves of Plucked String.

The string plucked by the finger shows a stage intermediate between these two. The initial curve is not so fully rounded as the bow wave, nor is it, owing to the width of the finger and the resistance of the flesh, so sharply kinked as the other, consequently the tone is less rich in higher harmonics, and is tolerably mellow.

As to the best point on the string to pluck, this is determined by the fact that the seventh and ninth harmonics are dissonant to the equi-tempered scale, as are many of the

higher members of the series, but these last are of such
feeble intensity that we need not trouble about them.
The Young criterion teaches us to pluck it at one-seventh
and one-ninth of its length from one end if we wish to
cut out these harmonics. We cannot use the plectrum at
both places, so we choose one-seventh for the position, but
the finger is wide enough to cover both these points, except
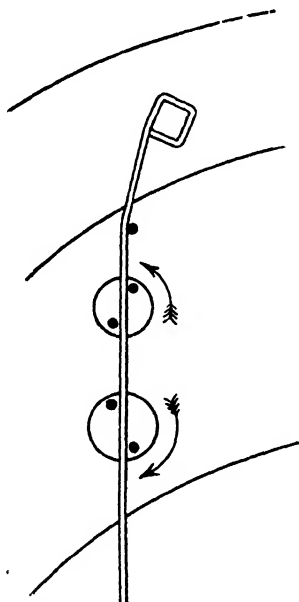on a long string.

FIG. 25.—Device for altering the Sounding Length of a Harp-string.

Space does not suffice to trace out the evolution of the
harp in all its forms. The harp would require a separate
string for each note of the musical scale, but in the orches-
tral form sufficient strings are provided for playing in one
key only (C♭ major). This saving of space and expense
is due to a device of Erard, which consists of little rotating
discs having two pins which grip the wire on being rotated
(Fig. 25). Two of these are applied to each string and

serve to shorten the sounding part of the wire; the upper disc raises the pitch by a semi-tone, the lower by a full tone. These are operated by seven pedals, each one controlling one of the seven lowest notes and all of its octaves, so setting the instrument for playing in other keys. By an ingenious system of levers a half-depression works the upper discs only, while a full depression of the pedal brings on the lower ones as well. It is possible to set any minor scale in one only of its forms, i.e. ascending or descending.

The case of the struck string presents many points of similarity to the plucked string. We should expect that a hard, sharply pointed hammer would produce a kinked wave and consequent note rich in upper partials, and that a soft rounded hammer would produce a hog's-back type of wave and a more mellow tone. In point of fact these expectations are realized, the main difference of quality between the note of the harp and that of the piano being due to the suddenness of the displacement of the piano wire. Under the comparatively slow motion of the finger a plucked string has time to adjust itself to the new forces before the finger lets go, and just before this happens the whole wire is momentarily at rest. But the blow of the hammer lasts only for a time comparable to the time of oscillation of the string. Photographs show that a piano hammer remains in contact with the wire for less than the time of one vibration before it flies back. Under these circumstances, the struck point moves in advance of more remote parts of the string, which may have only just started to move before the blow is finished and the struck point is beginning to return. This naturally complicates matters to an extent depending upon the force of the blow of the hammer. The force of the blow can be increased by increasing the mass of the hammer and by allowing it to

hit the wire at greater speed; these expedients are found to increase the intensity of the fundamental tone of the wire if the blow is applied, as it is in the piano, near one end of the string.

As a matter of fact, the striking point chosen is one-seventh of the length from one end. It was formerly thought that this had the effect of removing the objectionable seventh harmonic, but recent experiments have cast doubt on the validity of applying Young's law to the special case of a smartly struck string. It has been shown, for example, that though a string be hit at one-eighth of its length from one end, the eighth harmonic (four octaves above the fundamental) is still present in the resulting sound. In fact, the striking point seems to be so chosen that the amplitude of the fundamental is large (*vide supra*), and that the forced vibrations of the sound-board across which the wires are stretched is at a maximum, whereas the natural vibrations of the sound-board—those which are excited by tapping the board and which the maker desires to eliminate—are reduced to a minimum. On a modern piano the two or three wires to each note are stretched between pins on the frame, the sounding portions being limited each by two bridges, one set of bridges being on the frame itself, and the other set on the sound-board, which is thus set into forced vibrations by the strings, without having to bear the strain of the total tension on the strings, which is borne by the frame itself.

The wires are damped fairly rapidly, but on all except the upper treble a "damper" checks their vibrations when the keys are released, unless the "loud pedal" is used.

When the frames were of wood the total tension could not exceed 10 tons weight, but with the introduction of the iron frame loads up to 30 tons weight could be safely borne. A glance at the fundamental formula for the vibrating string

given earlier in this chapter will show that if the tension on a particular wire can be increased, its length can be reduced, a fact which has afforded to the manufacturer the opportunity of satisfying the present taste for *multum in parvo* by the construction of baby grands.

Another point which has to be considered is the graduation in size or hardness of the hammers. Unless the struck surface bears the same relation to the length of the string all the way up the scale, the timbre will vary in different ranges of the piano. The treble hammers are therefore sharp and hard, while those of the bass are heavily felted so that the fraction of the string touched by the hammer remains approximately the same throughout.

Much has been written on that mystic quality called "touch," but this must be relegated to the domain of fetishes. A discussion took place at the Physical Society of London in 1912 in which, after considering carefully the mechanics of the piano, the conclusion was reached that the only thing which the player can vary is the velocity with which the hammer strikes the wire. As this is the only variable on the player-piano, there is no reason why a mechanical instrument should not do equally well all that the most brilliant virtuoso can accomplish. The rest is merely psychological, the sight of the player, his actions, and the effect of his personality on the audience.

The sound produced by an unaided vibrating wire is but feeble, since it presents such a small surface to the surrounding medium of air. All stringed instruments except the harp possess a sounding-board or cavity whose function is to be set in forced vibration by the wires, and to transmit the sound from its larger surface into the auditorium. The sound-board is therefore of great importance on the piano. It must first copy the vibrations of the wires faithfully, and then transmit them to the air efficiently. For

this latter purpose the surface of the sound-board must be large, but this surface must vibrate, as far as possible, in phase as a whole, not with parts moving in opposite directions as is the case with the complex vibrations of the plates described in the preceding chapter. Such opposed movements would cause interference in the air between the vibrations transmitted by the out-of-phase parts. To secure the desired type of vibration it is essential that the sound should spread out rapidly from the bridges all over the sound-board, i.e. that the velocity of sound in the board should be great. This is secured by choosing a wood of great elasticity but of small density. Norway spruce best fulfils these conditions, for which the velocity of sound along the grain (15,000 feet per second) is nearly equal to that of steel. Unfortunately, as in all woods, the velocity across the grain is much lower—only one-quarter in Norway spruce—which tends to the formation of nodal lines along certain of the grains. The propagation of sound across the grain is aided by "ribs," strips of wood cut along the grain of the tree, and glued across the grain of the sound-board. Furthermore the sound-board is arched slightly under compression to give greater elasticity.

We come finally to the instruments of the violin family, of which we can treat the violin as the archetype, except when special mention must be made of the other members. The main point, acoustically speaking, which differentiates the violin from the harp and piano is that the loss of energy due to friction, which would bring the wire to rest, is made up by the energy supplied by the player through the bow, so that the sound may be sustained. The two statements made with regard to the plucked string apply equally to the bowed string, i.e. (1) if the bow is narrow compared to the length of the string the note will be richer in high harmonics

VIOLIN.

PLATE XV (Hawkes & Son)

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than if it is wide, (2) those harmonics are absent which would have a node under the bow. The dissonant high harmonics may be removed by bowing with a fairly wide bow whose centre is at about one-ninth of the length of the string from the bridge; the usual place is from one-seventh to one-fifteenth. If the bow is applied very close to the bridge, it will not cover any nodes except those of impossibly high harmonics, and the note will be metallic. This playing near the bridge (*sul ponticello*) is sometimes introduced by composers for special effect. Playing with the wood of the bow (*col legno*) has a similar result.

The action of the bow upon the string has been explained by Helmholtz and substantiated by experiments, as follows. When the bow is first pulled across the string it drags the latter in contact with it, until the restoring force in the string becomes too great for the friction to keep the two together, and the string comes to rest in its far position. Now it is a well-known fact that friction between surfaces having a relative motion is less than that between two adhering surfaces, so that the string now skids back past the bow although the latter is still moving forward, until the string has so far overshot its position of rest that the bow grips it again; thus the to-and-fro motion of the wire is maintained by a one-way motion of the bow. Increased pressure of the bow or greater velocity will increase the amplitude of the vibration, and so increase the intensity of the sound.

Let us now consider the complete instrument. The shape of the violin is too familiar to need description. Plate XV shows the relative disposition of the sounding parts. There are two sound-boards—the belly, to which vibrations from the strings travel via the bridge, and the back, which picks up the vibrations transmitted through the sound post **S**. Sandwiched between these is a resound-

ing cavity of air communicating with the atmosphere by the two ∫ holes at the waist, which also serve to give flexibility to the wood at this part. The bass-bar (shown dotted) is a longitudinal rib glued under the belly on the opposite side to the sound-post to strengthen this side. In order to give broad resonance to the complete range of the violin it is best to arrange that the natural fundamental tones of these three reinforcers—belly, back and air—should be fairly evenly distributed about the lower part of the range; their harmonics will then reinforce the upper part. The wooden sides may be tuned up by suitably fixing the sound-post between them. Dr. Fuhr, who has made a special study of this part of violin acoustics, recommends that the sound-post be so adjusted as to get good resonance to the notes (on the violin) of $a: c_1: e_1:$ while the air cavity is to look after the resonance of the notes G to g. The air is generally highly damped and exhibits broad resonance. The air cavity of good violins will resound to almost any fork in this range held over one of the ∫ holes.

Dr. Fuhr gives the following table showing the average natural notes of certain good instruments examined by him:

Instrument.	Lowest String Tone	Air Tone.	Lowest Wood Tone.
Violin	G	c	g
Viola	C	B (or A)	f
'Cello . . .	CC	GG	E

The sound-post is always placed behind and below one foot of the bridge; with the result that the bridge is not so much pushed and pulled by the vibration of the strings —which motion would indeed transfer little energy to the belly since the bridge is not rigidly connected to it—but

rocked about the supported leg, so that the belly and back
are made to vibrate like curved plates perpendicular to their
surface, thus exciting a larger mass of air. The sound-
post should be as wide and as elastic as possible to get good
transmission of the motion of the bridge to the back of the
violin.

The forced vibrations of the body of the violin add a
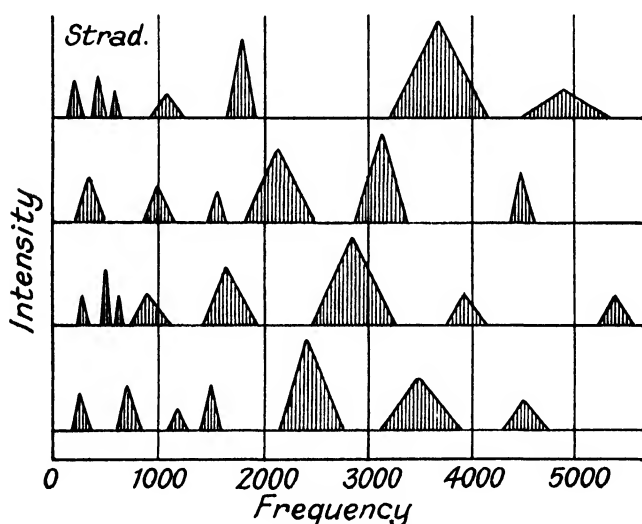certain richness to the tone of the strings. This point

Fig. 26.—Resonance Peaks of Violins (after Backhaus).

was thoroughly investigated by the late Prof. Barton and
his collaborators. They found that the record of the
vibrations of the belly showed a more complex wave-form
than that of the string, while the vibrations of the air
inside were more complex still. Hermann has applied
his "formant" theory to string instruments. Generally
speaking, the better the violin, the less complex is the wave-
form. Fig. 26 (after Backhaus) shows records of the
resonance peaks of (1) a Stradivarius, (2) an old Italian

violin, (3) a good modern violin, and (4) a violin of inferior quality: the first giving the less unevenly distributed resonances, most of the reinforcement lying between frequencies of 3000 to 4000.

Altogether the correct adjustment of the functions of the reinforcing elements of the string itself forms the main criterion which distinguishes a good violin from an indifferent one. The varnish does not appear to have any marked acoustical effect, although it slightly raises the natural tones of belly and back by giving them greater rigidity.

The harmonics between 2000 and 4000 vibrations per second are prominent in a violin; they are marked tones due to resonant vibrations of the body and the air in it. To these resonant tones much of the difference in quality between a violin, a viola and a 'cello playing the same note is to be ascribed.

To get good reinforcement the belly must be elastic and under tension like the sound-board of a piano. This is secured on the one side by the sound-post, and on the other by a bass-bar to which the wooden plate is glued while under tension in the act of curving it. The bass-bar thus gives both support and elasticity to this side of the violin.

Another factor which may reinforce or influence the tone of a bowed string is to be found in the other strings which are momentarily idle. For example, suppose d be played on the G string stopped at the requisite point, the d string being in tune with this will take up the vibration transmitted through the bridge. This sympathetic vibration is well known, so much so that sympathetic strings have been applied to a number of stringed instruments. This was the principle of the viola d'amore, which, besides the bowed strings, had other strings tuned to these and stretched beneath the finger-board to points lower down on the bridge.
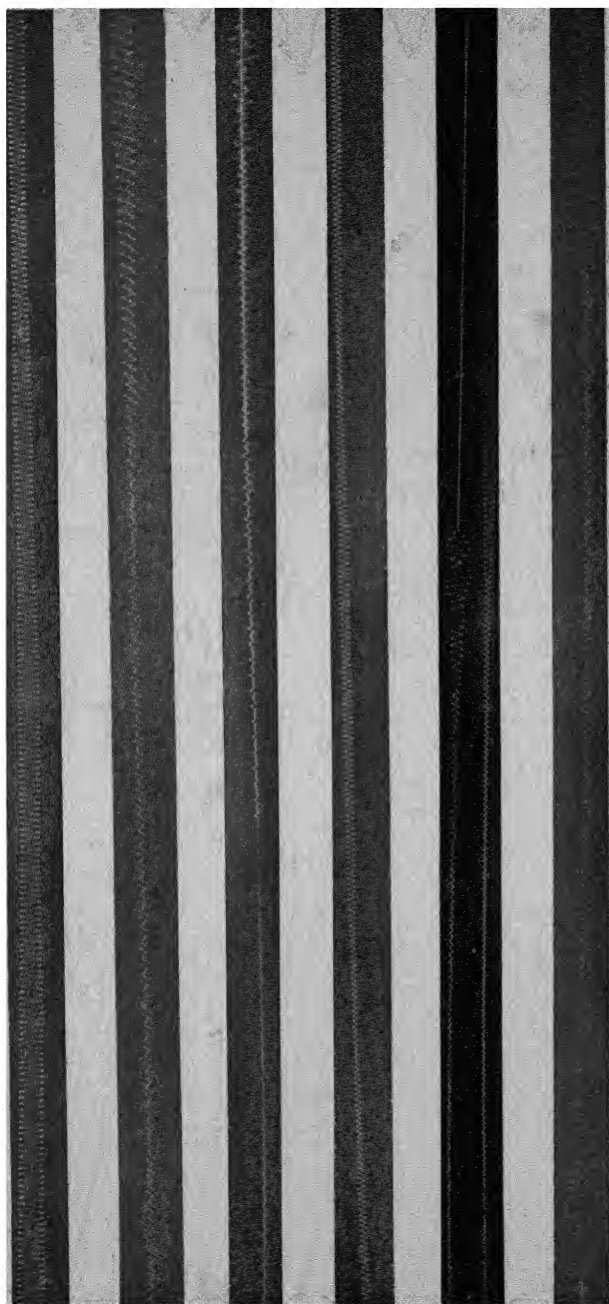
VIOLIN RECORDS (after Dr. Mary Browning).

PLATE XVI

They have also been tried on the piano (Steinway duplex scale), but the adjustment is too delicate to merit extensive use.

Plate XVI shows some records of the sounds from a violin taken by Dr. Mary Browning. The plucked note is found to contain more higher harmonics than the bowed note, probably because the first method gives an initially more jagged form to the string than the latter. The first two records show the note c_1 (512) bowed and pizzicato; the next pair shows the same on the note G (192). It is of interest to see at what rate one note dies down when another is produced, as this affects the certainty with which rapid passages can be executed without blurring. The fifth film shows the result of plucking the a and G strings successively; the sixth, of bowing the d and G in rapid succession. In each case there is a comparatively long period of overlap during which the two notes interfere with each other.

When the note played by bow or finger on one of the strings coincides with the fundamental or an important overtone of the wood or air one would expect a large reinforcement of the tone. In fact, if the resonance of the wood is too sharp, an undesirable effect takes place at a pitch in which the violin takes up so much energy that the control of the note seems to pass out of the player's hands. This note, which is very prominent on badly constructed violins, is known as the "wolf note" from its howling effect. Curiously enough, it does not seem to occur at the fundamental of the wood, but only at one of the higher overtones, though it is, of course, at a lower pitch on the viola and 'cello than on the violin itself. Records of the motion of the belly show a simple type of vibration of large amplitude at the wolf note, whereas at any other pitch the wood shows a smaller but more complex vibration.

The acoustical phenomena relating to the violin which we have described show what an important part the wooden structure of the bowed instruments plays in the tone-production. The violin is, in fact, a coupled system of at least four stages, viz. string, bridge, belly, back and air cavity. Each of these must be well constructed, and the couplings between them arranged to get the best effect. Very often the adjustment of the various parts improves with age and use. In fact, the improvement takes place automatically after the instrument has been played on many times, for the instrument is forced into efficient tone-production by the prolonged action of the player. There is room yet for a great deal of research toward the perfection of the bowed instrument.

The mute exerts a potent influence not only in reducing the intensity of the sound emitted, but in altering the timbre. The forked attachment—small for the violin, massive for the double-bass—loads the upper part of the bridge in such a fashion as to add considerable inertia to its movements. Of the two movements of which the bridge is capable, one, the push and pull of the bridge, is probably hindered more than the rocking movement about the foot which rests over the sound-post. This change will be sufficient to affect the quality of the notes, but the added load will also lower the natural tones of the structure, including the wolf note. This lowering will also change the quality. It is not merely the mass of the mute that matters, but also the position of that mass, i.e. whether the mute projects high above the bridge or lies close down upon it.

Though not an orchestral instrument, the strohviolin may be mentioned at the conclusion of this chapter as an unusual acoustic combination. A single string passes over a bridge which is pivoted in such manner that it can vibrate

only in its own plane and therefore transversely to the string. The bridge is prolonged below the breast of the instrument by a slender curved foot which ends in the centre of a metal diaphragm at the apex of a horn and imparts to it the vibrations of the string. The amplitude of the movement is magnified by the length of the foot, and intense sound-waves spread off from the large surface of the diaphragm. It is therefore a coupled system in three stages: string, membrane, and horn. The latter is made of wide angle so as to have no marked resonances, with the result that the air in it is set into forced vibration like the air in the body of a violin.

CHAPTER VII

ENSEMBLE

THOUGH we have now considered the acoustical pro-
perties of the individual constituents of the various
orchestral instruments in detail, our work would not be
complete without reference to the orchestra as a whole,
and its relation to the listener in a concert-hall.

It has been pointed out, in a number of instances, that
quality depends on the number and relative magnitude of
the partial tones in a complex note, also that particular
instruments are characterized by the emphasis of their
partial tones over a certain range of the scale, i.e. by their
formant. Without discussing the physiology of the ear,
one can state that our ears are able to pick out the partial
components of a complex note or chord, but that the rela-
tive phase between the components cannot be so detected.
The former fact is self-evident to any musician, whose ear
has been trained. The difference between persons in this
respect is only a matter of training or the lack of it. To
put the second statement in concrete form: suppose a
fundamental and its octave are sounding together. These
two partial tones may be in step in the sense that every
alternate compression in the octave coincides with a com-
pression in the fundamental, or they may be out of step
when the alternate pulses of the octave lag behind the pulses
of the fundamental. The resultant waves differ physically
in the two cases, but many experiments have shown that
the ear is not cognizant of this difference.

In spite of the ear's powers of analysis we seem to retain the capability of assigning a particular partial tone in the mass of tone colour from a small orchestra to the instrument which is producing it, except possibly in a full *tutti*. The spacing of the various instrumentalists over the concert platform is an aid to us in this determination; for, using our two ears together, we are able to detect from what direction the partials comprising a particular note come. If a source of sound lies to one side of the head, there will be both a difference in intensity and in phase between the waves received by each ear from the source. On the far side the intensity will be less because this ear lies in the "shadow" of the head, while the waves will arrive later than those at the nearer ear, owing to the longer path traversed. From one or other of these indications the brain is able to locate the direction from which the sound comes, and this faculty helps in allotting the various partials to the sources producing them.

The discrimination is also aided by the fact that the instruments are mostly starting and finishing a note at different instants, while each fresh note reveals its instrument by the peculiar mode of attack; the violins, more particularly the 'cellos and basses, by the scraping noise which precedes the bite of the bow on the string; the flute and brass by the tonguing or *ictus*; the reeds by rush of wind before vibration sets in; all these, though cut down to a minimum by a good player, may be heard by the trained ear.

Then there are the transitions such as those illustrated on Plates XI and XVI, and the rapid damping of unsustaining instruments like the harp and timpani.

Besides these differences, it has been suggested that our appreciation of individual instruments in an ensemble is aided by the varying powers which instruments possess of

radiating sound in certain directions. Just as a vertical arc lamp radiates more light in a horizontal plane than in other directions, so it has been found that a violin sends out more sound in the plane of the belly than in directions more nearly perpendicular to the strings. The wind will, of course, radiate most sound from the bell, i.e. inclined downwards in the wood-wind, and nearly horizontal in most of the brass. The suggestion is, that as we are used to listening in a certain direction relative to the position in which the player holds his instrument, that this is one of the factors by which we recognize its quality in the orchestra.

Incidentally, this radiating power varies with the pitch of the partial tone, and might vary the apparent quality of the instrument when the listener varies his position horizontally or vertically in regard to the player. At any rate, whether this refinement of perception be accepted or not by the psychologists, it should be apparent that the theory has advanced far beyond the original one of Helmholtz, that quality is determined solely by the number and relative magnitude of partial tones.

It is, no doubt, in accordance with this idea, i.e. that the audience will be able to pick out the parts in an ensemble, that some books on orchestration insist that the harmony for each branch of the orchestra, strings, wood-wind and brass, shall be complete in itself. This cannot be justified acoustically. This might do for the strings, since they might, speaking loosely, be said to possess similar tone-colour. But if a note in the harmony be wanting, it cannot matter acoustically whether that note is produced by trumpet or flute, provided it be played at the requisite intensity to balance the remaining parts of the harmony.

It is in the latter desideratum that the difficulty arises. If an instrument has a high and prominent "formant," like the oboe, for example, it will be difficult for the player to

introduce sufficient of its fundamental into the middle of a chord from the orchestra without bringing out objectionably incisive overtones. Such an instrument has to be kept in the main to solo or *tutti* work. Nor do wind instruments combine well when duplicating passages in unison.

The strings being less powerful in tone, a number may be combined together in the orchestra, and the mass will smooth out the idiosyncrasies of the individual instruments. But two clarinets playing together will only serve to exaggerate the differences in player and instrument. This was perhaps at the bottom of Cherubini's famous *bon mot* that nothing could be worse than only one flute in an orchestra —except two flutes. If it were feasible to have twenty flutes or twenty clarinets in an orchestra, this trouble would not arise; though others might!

Locality-discrimination has also to be exercised when one instrument or a voice has a solo, and the rest of the orchestra accompanies. It is obvious that the solo instrument must, both in its fundamental and partials, overpower the mass of tones in accompaniment, particularly if the solo be a middle part, for the ear tends to pick out the highest notes in a succession of chords. Why the ear—particularly the untrained ear—should do this is a matter for the psychologist, though one may suggest it is due to the fact that every person in his or her youth learns to sing the top part.

The voice lends itself well to orchestral accompaniment whether alone or in chorus. This is because the voice in itself is a species of wind instrument, most closely related to the horn, with its own formant due to the nasal resonances and other cavities in the head. The voice can be tolerably imitated by a stop of reed pipes on the organ. At its best this is really a "vox humana" stop, at its worst it has been likened to a quartet of human goats. The organ is not

so successful in combination with the orchestra. As a solo instrument it lacks the expression which the wind players can give to their individual instruments, while as a background to the orchestra it is apt to spoil, and be spoiled by, the orchestral wind.

The question arises as to what is æsthetically the best "mixture" of tones in a chord. Now that tuning-forks can be maintained electrically at any requisite intensity, it should be possible to mix the tones of a concord of forks in any proportion, in order to find an ear-satisfying colour. It is well known that the ear soon tires of a pure tone, such as that of the flute. On the other hand, there are certain reed stops on organs which would be intolerable if played alone. Prof. Miller suggests that an ideal instrument would have partial tones whose intensity would increase as the pitch went up.

Attempts have been made to correlate visual colour with sound, or to create an independent form of colour music by the projection of a series of tints upon a screen. As light consists of waves propagated at great velocity through the ether, each tint having a definite wave-length, it is natural that the mind should draw an analogy between a melody as a series of notes each of definite wave-length, and a corresponding · series of colours. The analogy is strengthened by the fact that the visible spectrum almost covers an octave; the frequency of the extreme violet radiations being almost double that of the deepest red. If the colour range be divided up into twelve semi-tones corresponding to the divisions of an octave in the key of C major, it is possible to reproduce a melody by successive projection of these colours on a screen, provided the melody does not exceed the compass of an octave. Mr. Rimington has constructed colour organs consisting of such tints at "semitone" intervals which can be projected on a screen, while

composers have endeavoured to use such an organ as an additional instrument to an orchestra, or to give a colour concerto accompanied by the orchestra.

Although the analogy is a close one from the physical point of view, yet it fails in an important respect when applied to our perceptive faculties for the two sensations. Whereas the ear has the power of analysing a complex note into its constituents, the eye cannot analyse a mixture of colours. While a major chord on C appears to the ear to consist of the tones C, E and G, a mixture of two colours, red and yellow, is not perceived as a "chord" of red and yellow, but as "orange," an intermediate tone. The impossibility of constructing harmony capable of intellectual appreciation on the colour scale has rendered abortive the attempts at colour music made up to the present.

We cannot conclude this chapter on the interaction between an orchestra and a listener without considering the acoustics of the auditorium in which they are placed. The desiderata in this respect are that the sound in the room shall reach an auditor in any part of the room at suitable loudness and distinctness and without undue reverberation, echoes, "dead spots," etc. Reverberation and echoes can be largely controlled by suitable covering on the walls and ceiling. The non-scientific hearer usually groups both effects under the general name of echoes. By reverberation is meant the sound which is heard during the time which elapses between the shutting off of a source of sound and the instant at which it can no longer be heard in the room. In listening to a band in the open air this time is a small fraction of a second, being in fact the time taken for the pulse last emitted to travel from the source to the listener; but within closed walls the sound continues to ricochet from wall to wall for perhaps two or three seconds, and will overlap that from the next note produced

by the instrument. Fig. 27*a* is intended to represent the rise and fall of loudness in a room, due to successive starting and stopping of a single note from a musical instrument. This loudness gradually rises to a maximum as the energy sent back to the hearer by successive reflections is added to the energy which comes to him directly from the source. The maximum level of loudness shown by the top of the curve represents the balance which is soon reached between the energy supplied by the source and that withdrawn from the room by transmission through the walls and absorption in the materials composing them.
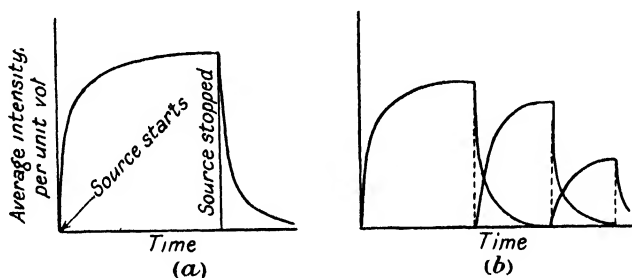
Fig. 27.—Rise and Fall of Loudness in an Auditorium.

When the source is stopped the curve drops steeply, for the main supply of energy is cut off, but the loudness does not instantly drop to inaudibility, because "bundles" of energy are still flying about between the walls until they are finally absorbed. The time taken for this to happen is the time of reverberation. It is evident that if it takes a long while for this energy to be absorbed, i.e. if the room is too reverberant, a succession of notes from an orchestral instrument will appear as a blur owing to the overlapping reverberations.

Fig. 27*b* represents in the same way the smudgy rendering due to a succession of notes—a solo, in fact—played in such a room. Though this defect of excessive reverbera-
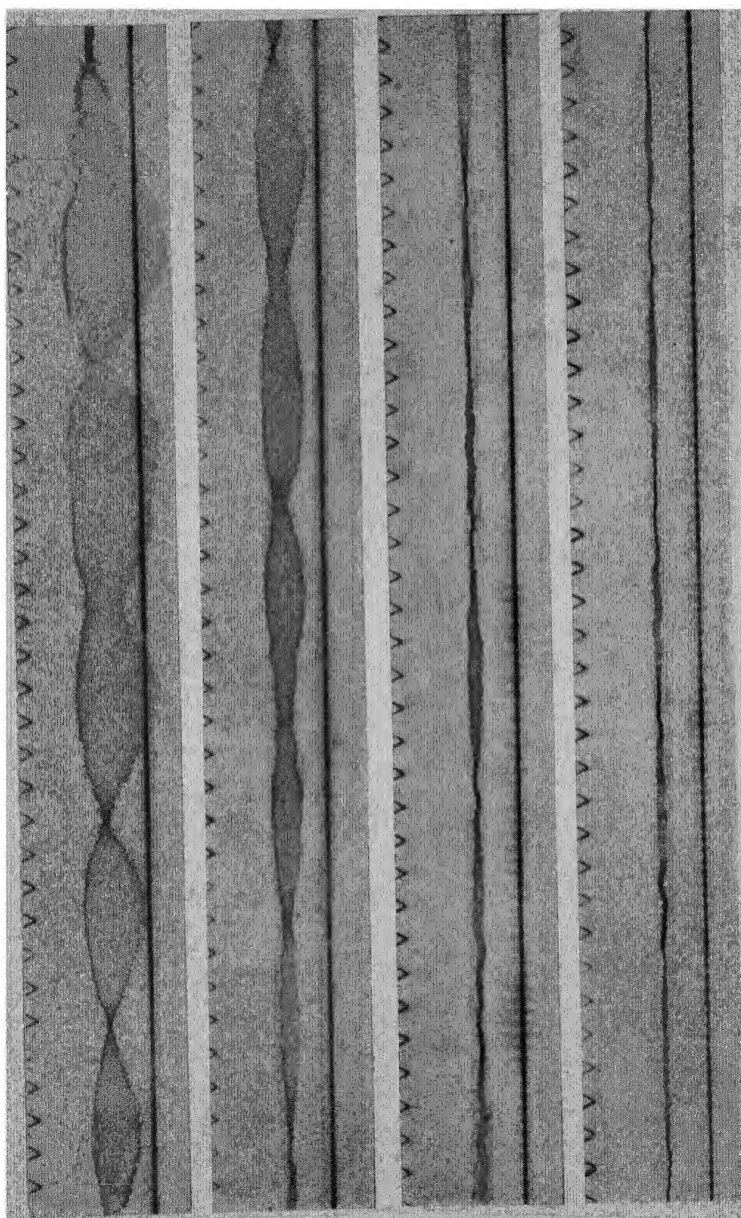
ECHO OF BELL IN COLOGNE CATHEDRAL (after Trendelenburg).

PLATE XVII

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tion is the more usual, yet the opposite effect may arise. If the sound is absorbed too readily, the energy will be cut off almost directly the instrument ceases to emit it. Such a building will give a "dead" impression and a very clear-cut rendering will result. This, though desirable for speech, may be overdone in the case of music; moreover, it will be difficult for a soloist to fill the room with a volume of sound, for the rapid absorption by the walls will make it difficult for him to keep up the general level of loudness in the room without over-exertion.

In halls of very large volume, such as the Albert Hall in London, there may be a distinct gap between the first sound reaching a hearer from a sharp pulse such as a drum-tap, and the first reflection of the same sound; then one hears an echo. Plate XVII shows an interesting case of an echo in Cologne Cathedral examined by a microphone whose response is represented by the swelling out of the shaded line; the photograph is to be read as a continuous strip from *right to left* down the Plate. The top right-hand corner represents the instant at which the great bell which chimes the hour was struck. Shortly after the microphone picks up the direct sound which waxes and wanes five times in succession. This is followed at an interval of 1·8 second by the echo of these five sounds, and later in the photo a second echo can just be seen.

Deadness is favoured by small volume and highly absorbent walls, for the sound is then absorbed in a series of rapidly recurring reflections, and *vice versa*. Statistics have been compiled which show how the absorbent area of the walls must be increased to keep pace with an increase of the volume of the concert hall, in order to satisfy musical criticism criteria.

The planning of concert halls and the correction of existing halls consist first and foremost in making the

actual time of reverberation agree with this optimum time for a given number of auditors. As the usual difficulty is to find materials sufficiently absorbent for the purpose, proprietary articles have been put on the market, having strongly absorbent properties, the most convenient of which can be obtained in the form of plaster to apply to the wall. For reducing "deadness" glazed brick or hard stone lining may be used. Particulars of these substances are given in articles or books devoted to the acoustics of buildings. As substances absorb tones in different parts of the musical scale to a different extent, the rendering of a work may involve a certain amount of distortion due to the properties of the building itself. Uncertainty as to the probable number of the audience, whose clothes provide a good deal of the absorbent material in a concert room, may be allowed for by putting down thick carpets or hanging up thick curtains, to be drawn aside if expectations in the number are more than justified. A finer grading may be attempted by having regard to the character of the music. The organ requires more absorbent surroundings than a staccato instrument such as a piano, while a singer will have trouble in getting his words understood if the hall is as reverberant as an orchestra would require it.

Having determined the amount of absorbent material required for satisfactory listening, the question arises as to where it should be placed. If the building has a large unbroken curved surface, this is liable to concentrate sound at one point to the detriment of other parts. Further, if this surface lies at the back of a large hall, it may cause an echo to a listener near its focus if there is a considerable difference of path-length to be traversed between the direct sound and that reflected in this big "mirror." In this case the remedy is to put the absorbent on the offending surface, if it can be found. The same consideration tells
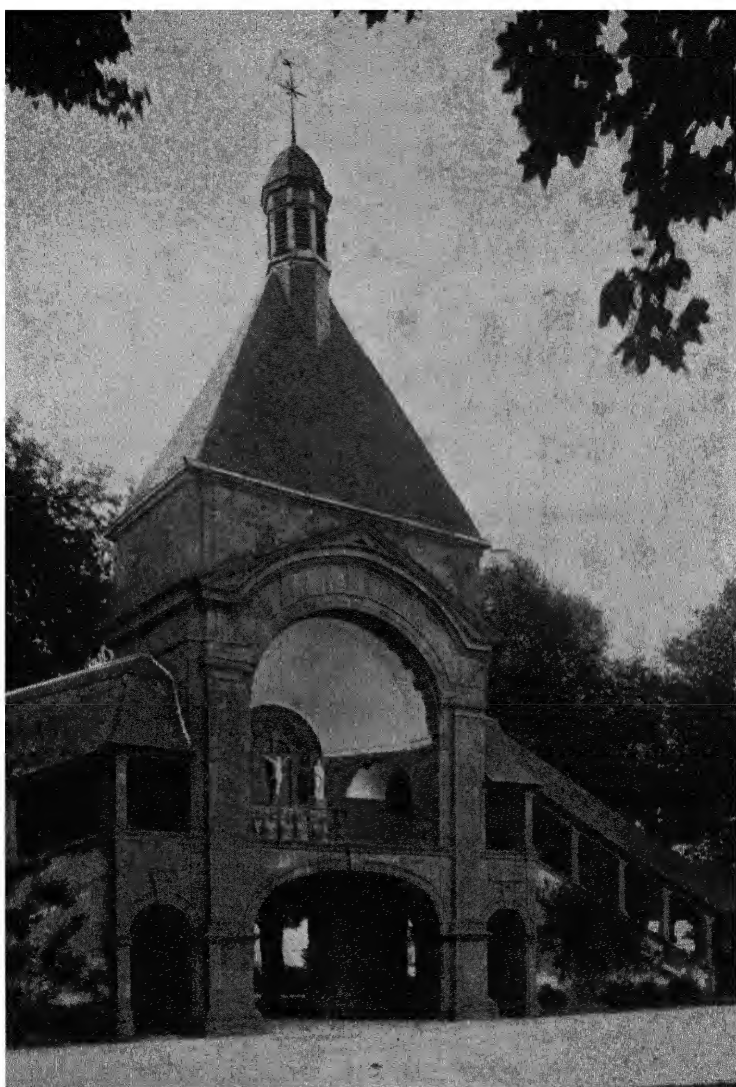
THE SCALA SANCTA OF STE ANNE D'AURAY

PLATE XVIII

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us, if we are concerned in the building of a concert hall, not to allow the architect to put in an excessively high roof, which is bound to produce undesirable echoes unless its surface be broken up or covered with absorbent.

On the other hand, it is best to place the orchestra on a bare platform, where they will be backed by a good reflecting surface, for this will act on the source of sound in the same way as in Plate I, p. 17, where a reflected wave starts out on the heels of the direct wave, and so soon afterwards that it *adds* its energy usefully to the direct sound reaching the audience. For this purpose the orchestra should be
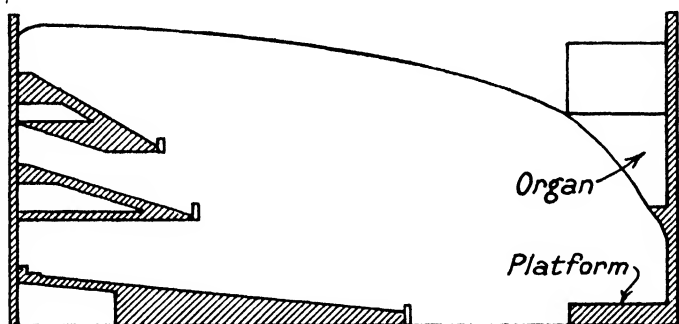
Fig. 28.—Elevation of Pleyel Hall.

placed in a shell-shaped recess or niche, backed by hard material, stone or wood sheathing, and any absorbent required for reducing reverberation must be placed round the audience, not round the orchestra. Such a niche has the excellent property of sending out a beam of sound to the audience. As an example of the—probably fortuitous —application of this principle, we may cite the Scala Sancta of the church of Ste Anne d'Auray in Brittany (Plate XVIII). In the celebration of open-air mass the voice of the priest standing within the central niche carries to great distances in the direct line.

In the concert hall of the Salle Pleyel at Paris, M. Leon

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has tried to use the ceiling also for the sending out of sound. An elevation of the hall along its length is shown in Fig. 28. It will be appreciated that the low sloping ceiling just in front of the platform catches the sound-waves passing up and reflects them towards the galleries in a horizontal direction. Another point about this hall is that the organ is placed behind this low ceiling, and its sound comes out of a grille above the orchestra (Plate XIX). This was done to prevent the spoiling of this ceiling mirror by breaking up its surface with an exposed organ. This enclosing cannot but affect the transmission of sound from the organ detrimentally, but the arrangement of a single grille opening near the ceiling is to be preferred to the two side grilles so common in motion-picture theatres, from which the "great" and "swell" blare alternately at the audience from either side.

We may conclude with a word on the size of the orchestra adapted to halls of various dimensions. The table below (after Heyl) gives the recommended number of players in terms of the size of the hall. It is of course assumed that the balance of tone between the instruments is maintained in the selection of the instruments to make up the number.

Volume of Hall in cubic feet					Number of Instruments
50,000	.	.	.	.	10
100,000	.	.	.	.	20
200,000	.	.	.	.	30
500,000	.	.	.	.	60
800,000	.	.	.	.	90
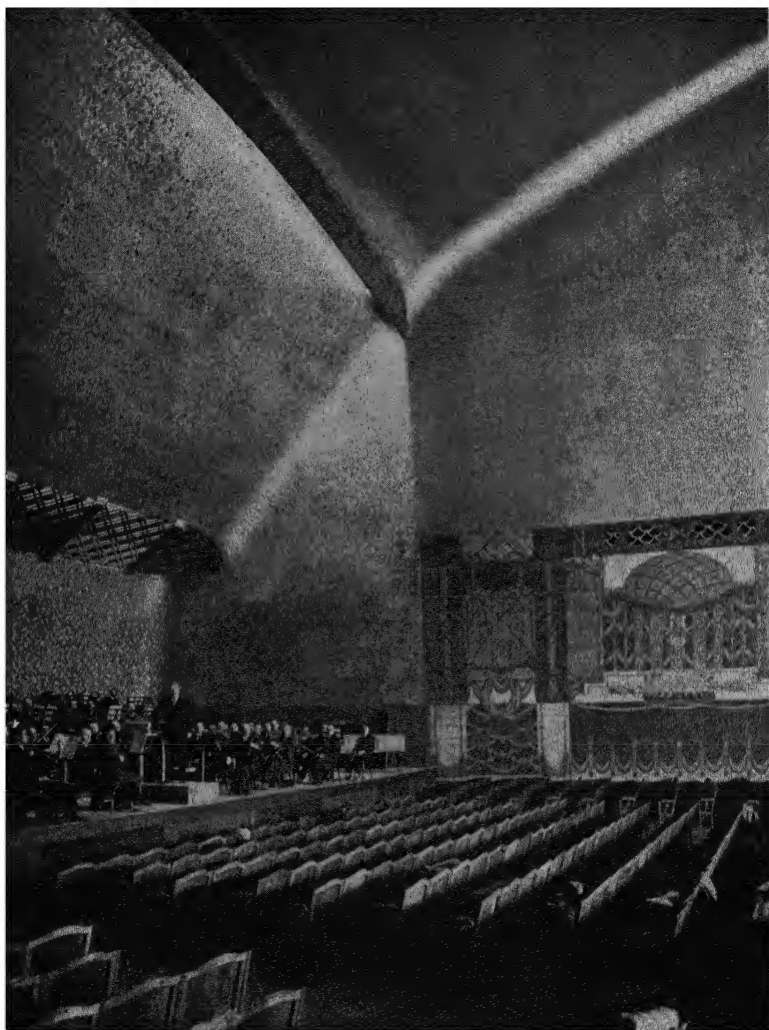
PLATFORM OF PLEYEL HALL.

PLATE XIX

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APPENDIX

THE THEORY OF FINGERING AND CROSS-FINGERING ON THE WOOD-WIND

We first suppose the displacement ξ of the air to be uniform over any section of area S, and put $X = \xi S$, where X is a sort of volume displacement. Then the velocity produced by an applied pressure excess p can be written $d\xi/dt$ or $\dfrac{1}{S}\dfrac{dX}{dt}$, which for brevity we will write $\dot{X}/S$. When p is alternating $(= Pe^{\iota\omega t})$ we define the impedance Z as the real part of $p/\dot{X}$, i.e. the ratio of the applied pressure to the volume velocity which it produces.

STOPPED PIPE.

The pressure amplitude of stationary waves along the pipe at resonance is given by $p = Pe^{\iota\omega t}\cos\left(\dfrac{\omega x}{c}\right)$, where the pipe is stopped at $x = 0$ and open at $x = L$. ($c =$ velocity of sound; $\omega = 2\pi \times$ frequency.) The equation connecting "displacement" with "pressure" is

$$\varrho\frac{d^2\xi}{dt^2} = -\frac{dp}{dx} = +\frac{\omega}{c}Pe^{\iota\omega t}\sin\left(\frac{\omega x}{c}\right).$$

So that $\dot{X}$ can be found by integration with respect to time to be

$$\frac{S}{i\varrho c}Pe^{\iota\omega t}\sin\left(\frac{\omega L}{c}\right).$$

The effective impedance of the pipe due to the stationary waves in it is that at the open end where it is "transferred" to the atmosphere or to whatever acoustic system is attached to the end. Putting therefore $x = L$, we find

$$Z = -\frac{\varrho c}{S}\cot\left(\frac{\omega L}{c}\right).$$

The resonant frequencies are given by putting $Z = 0$.

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THE THEORY OF FINGERING

The equation of the stationary waves is now

$$p = P e^{i\omega t} \sin\left(\frac{\omega L}{c}\right).$$

Proceeding along similar lines we obtain $Z = \dfrac{\varrho c}{S} \tan\left(\dfrac{\omega L}{c}\right)$.

HOLE.

If A is the area of the orifice, the equation of motion of the mass m of air in the orifice is:

$$\frac{m}{A^2}\ddot{X} = P e^{i\omega t}.$$

Integrate, and $\dfrac{m}{A^2}\dot{X} = \dfrac{P}{i\omega} e^{i\omega t}$, whence $Z = \dfrac{\omega m}{A^2}$. But $m =$ A$l\varrho$, where l is the effective length of the orifice in the direction of vibration, and $\acute{A}$ is its area of cross-section.

$$\therefore \ Z = \frac{\omega \varrho}{\kappa} \text{ if } \kappa \text{ (conductivity)} = \frac{A}{l}.$$

One Hole open on a Flute, all others covered.

When the sound-waves pass down the flute from the mouthpiece, they have two courses open, one via the hole to the outer air, one along the remaining tube. This will be the case of an open tube of length L from embouchure to centre of hole, followed by another open tube of length l from hole to open end, both tubes having area of cross-section S. Then we must reckon the "admittance" ($= 1/Z$) of the hole (of conductivity κ) as being equal to the sum of the admittances of these two open pipes:

i.e.

$$\frac{\kappa}{\varrho \omega} = \frac{S}{\varrho c}\left\{-\cot\frac{\omega L}{c} - \cot\frac{\omega l}{c}\right\}$$

or

$$\frac{\kappa}{kS} = -\cot kL - \cot kl \quad . \quad . \quad . \quad . \quad (1)$$

where

$$k = \frac{\omega}{c} = \frac{2\pi n}{c}.$$

Clarinet with one Hole open, all others closed.

The admittance of the open hole is to be treated as equal to the sum of those of a *stopped* pipe of length L from hole to reed, and of an open pipe of length l from hole to bell, i.e.

$$\frac{\kappa}{kS} = \tan kL - \cot kl \quad . \quad . \quad . \quad . \quad (2)$$

Several Holes uncovered.

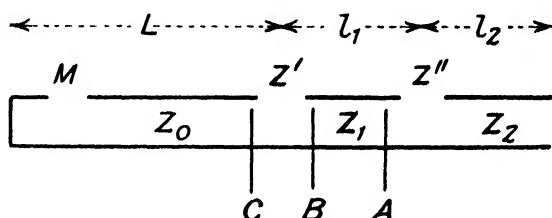If we adhere to the strict theory, this leads to a more or less complex arrangement of continued fractions.

Fig. 29.—Flute Impedance Diagram.

Consider the case of two open holes of impedance Z' and Z'' respectively, between three tubes of impedances Z_0, Z_1 and Z_2. Since Z'' and Z_2 are in parallel, the net impedance from the open end up to the point A is:

$$\frac{1}{\frac{1}{Z''} + \frac{1}{Z_2}}$$

Up to the point B the impedance is:

$$Z_1 + \frac{1}{\frac{1}{Z''} + \frac{1}{Z_2}}$$

At C, reckoning Z' in parallel with this, it is:

$$\frac{1}{\frac{1}{Z'} + \frac{1}{Z_1 + \frac{1}{Z''} + \frac{1}{Z_2}}}$$

Giving the total impedance up to the mouth, M:

$$Z_0 + \cfrac{1}{\cfrac{1}{Z'} + \cfrac{1}{Z_1 + \cfrac{1}{Z'' + \cfrac{1}{Z_2}}}}$$

This is zero if

$$\frac{1}{Z'} + \cfrac{1}{Z_1 + \cfrac{1}{Z'' + \cfrac{1}{Z_2}}} = -\frac{1}{Z_0}.$$

or

$$\frac{1}{Z'} = -\frac{1}{Z_0} - \cfrac{\dfrac{1}{Z''} + \dfrac{1}{Z_2}}{Z_1\left(\dfrac{1}{Z''} + \dfrac{1}{Z_2}\right) + 1}$$

Inserting the appropriate impedances:

$$\frac{\kappa'}{kS} = \left\{\begin{matrix} -\cot kL \\ +\tan kL \end{matrix}\right\} - \cfrac{\dfrac{\kappa''}{kS} + \tan kl_2}{\cot kl_1\left(\dfrac{\kappa''}{kS} + \tan kl_2\right) + 1}$$

the upper value of $-\dfrac{1}{Z_0}$ being taken for the flute, the lower for the clarinet.

When we can reckon that Z'' and Z_2 do not impede the motion, so that $Z'' = Z_2 = O$, the equation reduces to $\dfrac{1}{Z'} = -\dfrac{1}{Z_0} - \dfrac{1}{Z_1}$ the same as for one hole, when the smaller length of tube terminates at the more distant hole. This is generally justified by the experiments of Steinhausen.

Application of Theory to Fingering and Cross-Fingering.

Equations (1) and (2) can be solved graphically for k. We will take the case where the first 2 holes are closed, the rest open; thus M ● ● ○ ○ ○ ○. On my flute, the

distance L from cork to hole 3 is approximately five times the distance l between the centres of holes 3 and 4. We plot the co-tangents vertically against $\dfrac{1}{\text{angle}}$ $\left(\text{in } \dfrac{1}{\text{radians}}\right)$ horizontally, but the scale of the one is five times the scale of the other, and its vertical axis is reversed, i.e. the two curves are cot (5θ) and cot $(-\theta)$ (Fig. 30).
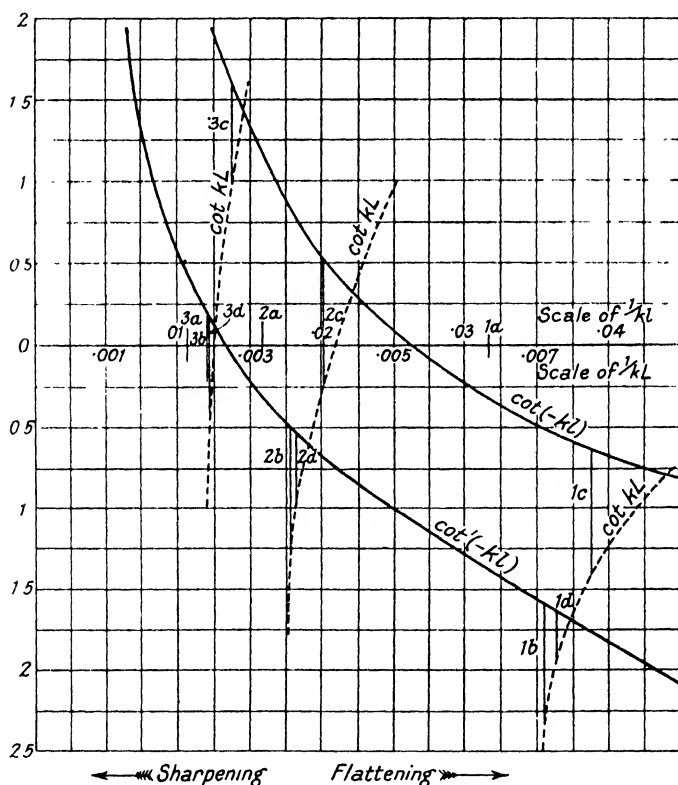

FIG. 30.—Fingering Diagram (Flute or Oboe).

Having determined, from the dimensions of the hole 4, that $\kappa = 1$ cm., while the inner cross-section S of the tube at this point is $1 \cdot 35$ sq. cm., we get $\dfrac{\kappa}{S} = 0 \cdot 75$ approx.

Formula (1) in the form:

$$\frac{\kappa}{k\mathrm{S}} = \cot\left(-\,kl\right) - \cot k\mathrm{L}$$

indicates that we must set this intercept (= 0·75 on our vertical scale) vertically between these two curves. These lines are marked *b* on the figure and we must see what value
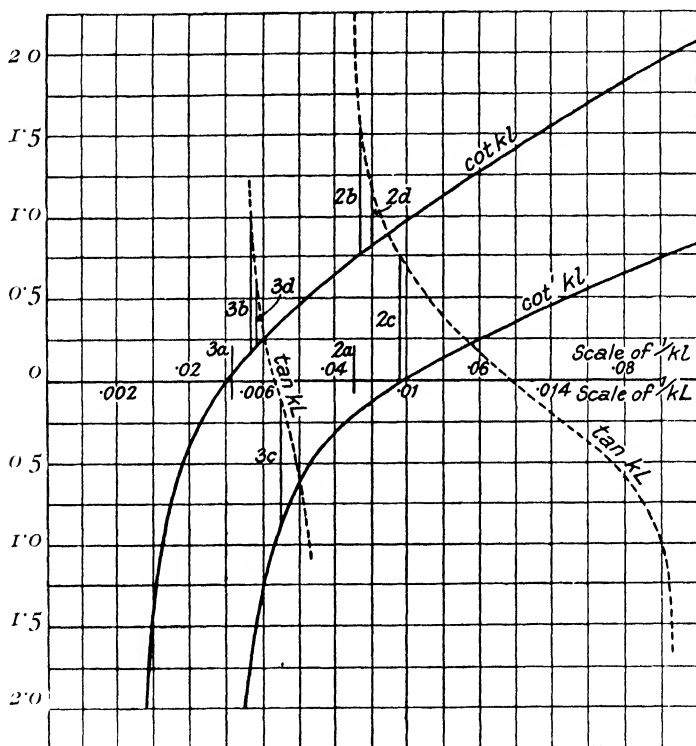
Fig. 31.—Fingering

of the horizontal scale lies under them. This gives us the corresponding value of $1/\theta$, and if we note that these values of θ are really *values of* $k\mathrm{L}$, we can, by multiplying the corresponding values of θ by L (= 30·5), find what values of $\frac{1}{k}$ correspond to these intercepts, and thus find the partial tones given by a flute so fingered. In the next table the

k's for the first three partials given by the lines (b) are set out.

Now suppose we want to flatten these notes by a semi-tone or less, there are two possible methods. (1) Cover hole 4 (cross-fingering), and so approximately double
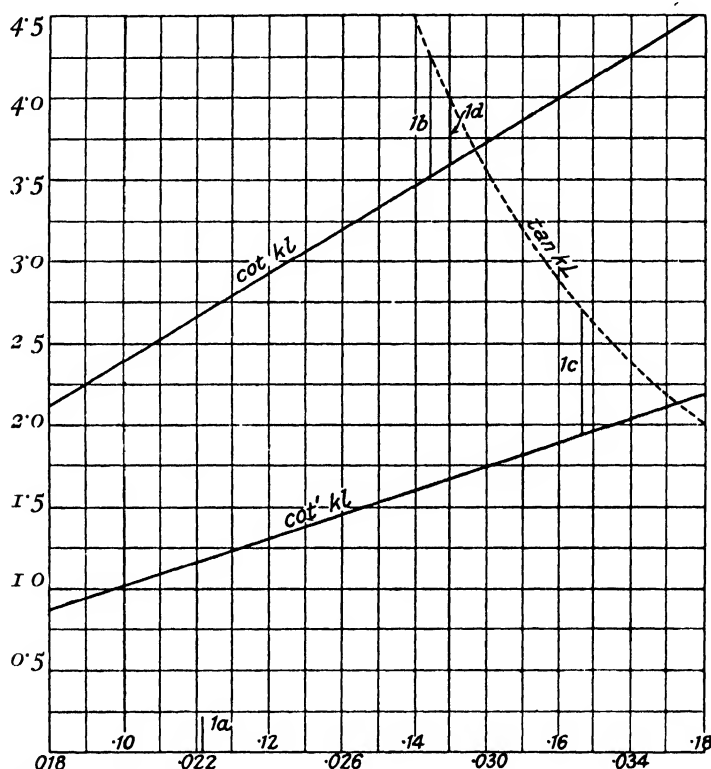
Diagram of Clarinet.

column l. For this we plot a new cot ($- l$) twice the scale of the original, and put in our same intercepts. The new cot' ($- l$) is shown by the lower continuous line on the figure, and the intercepts are marked c. (2) Partially cover hole 3 with the finger, and so reduce its conductivity. Intercepts of half size (d) have been put on the figure to show this effect.

The actual and theoretical values of k and the frequency for the first three harmonics are compared with the actual (for the first and second) on the flute concerned.

FLUTE.

Fingering.	k (from diagram).			n (theoretical).			n (actual).		
	1	2	3	1	2	3	1	2	3
a ● ●] . .	·100	·200	·300	540	1,080	1,620	—	—	—
b ● ● O .	·092	·184	·268	487	993	1,430	480	980	—
c ● ● O ● O	·085	·163	·238	450	880	1,270	460	900	—
d ● ●'O . .	·090	·179	·264	476	965	1,420	470	950	—

The cases marked (*a*) are the purely theoretical ones where the tube is cut off completely at the third hole, and is there supposed to terminate in an open end of infinite conductivity; they correspond to places on the diagram where cot $kL =$ infinity. The corresponding first three harmonics shown in the table are truly harmonic (ratios 1 : 2 : 3), whereas, for the others, neither theoretically nor actually is this relation exactly fulfilled.

For the clarinet we plot out cot θ and tan θ to the necessary scales, and put in intercepts as before, which are shown in Fig. 31.[1]

CLARINET IN A.

Fingering (as on flute).	k (from diagram).			n (theoretical).			n (actual).		
	1	2	3	1	2	3	1	2	3
a	·052	·156	·260	280	840	1,400	—	—	—
b	·047	·150	·232	259	810	1,250	255	780	—
c	·043	·145	·205	231	784	1,130	245	770	—
d	·046	·146	·224	247	790	1,280	250	775	—

[1] The reader should note that for convenience the right half of the figure has been lowered. He should imagine this half raised up about half its own length, until the horizontal scales correspond.

Again the relation $1 : 3 : 5$ between the frequencies of the partial tones is only approximately filled.

Note that the impedance of the cone is a complex function, but when the cone has a small angle and a side hole open, it virtually consists of two *open* pipes, so that the same diagram as for the flute can be applied to the oboe, provided that the value of S be measured opposite the hole.

Impedance of Resonator.

As xylophones and similar percussion instruments are provided with tuned resonators consisting of a closed pipe with an upper orifice of given conductivity, the frequency can be calculated by putting the total impedance, orifice + closed pipe, equal to zero, i.e.

$$\frac{\kappa}{k\mathrm{S}} = \tan kl.$$

The problems have been treated as those of determining frequency from given conditions, but obviously the same methods can be applied to the converse problem of fixing the position and designing the note-holes.

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